DESIGN AND CONSTRUCTION
OF HYDRAULIC FLUME AND BACKWATER EFFECTS
OF SEMI-CIRCULAR CONSTRICTIONS
IN A SMOOTH RECTANGULAR CHANNEL

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by

H.J. OWEN

Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

Progress Report No. 2

TO: K. B. Woods, Director   January 21, 1960
        Joint Highway Research Project

FROM: H. L. Michael, Assistant Director   File: 9-8-2
        Joint Highway Research Project   Project: C-36-62B

        Attached is a progress report entitled, "Design and Construction
of Hydraulic Flume and Backwater Effects of Semi-Circular Constrictions
in a Smooth Rectangular Channel". This report has been prepared by
Mr. H. J. Oxen, graduate research assistant on our staff, under the
direction of J. W. Delleur. Mr. Oxen also utilised the report as his
thesis in partial fulfillment for the requirement of the M.S.C.E.
degree.

        The material reported in this report is a summary of the progress
that has occurred on the Hydraulics of Arch Bridges Project which is
being conducted in cooperation with the Indiana State Highway Department
and the U. S. Bureau of Public Roads. Copies of this report will also
be distributed to the State Highway Department and the Bureau of Public
Roads for their review and comments.

        The report is presented to the Board for the record.

                                            Respectfully submitted,

                                            H. L. Michael
                                            H. L. Michael, Secretary

HLM:kmc

Attachment

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Progress Report No. 2

Design and Construction of Hydraulic Flume
and
Backwater Effects of Semi-Circular Constrictions in
A Smooth Rectangular Channel

by

H. J. Owen
Graduate Assistant

ACKNOWLEDGMENTS

The author wishes to acknowledge and thank Dr. J. W. Delleur whose guidence and help was invaluable. Appreciation must also be expressed by the author to his wife Pat. Without her help, the preparation of the manuscript would have been difficult.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

ABSTRACT

H. James Owen, M.S.C.E. Purdue University, January
1960. "Design and Construction of Hydraulic Flume and
Backwater Effects of Semi-Circular Constrictions in a
Smooth Rectangular Channel". Major Professor: Dr. J. W.
Delleur.

The purpose of this study was to investigate the
hydraulics of semi-circular constrictions in smooth rect-
angular channels. To this end a large part of this work
consisted of the design and construction of a hydraulic
flume 64 feet long and 5 feet wide. This study is part
of a general program sponsored by the State Highway De-
partment of Indiana and the U.S. Bureau of Public Roads
at Purdue University on the Hydraulics of River Flow Under
Arch Bridges.

INTRODUCTION

This project was initiated in the Hydraulics laboratory
of Purdue University by the Indiana State Highway Department
in cooperation with the U.S. Bureau of Public Roads, to study
the backwater and regain phenomena associated with arched
constrictions such as are presented by arch bridges. The
problem was approached by both theoretical analysis and model
study. The preliminary analysis of the general problem was
made by Mr. S. T. Husain[1] . Mr. A. A. Sooky[2] derived
exact and approximate equations for the two dimensional sharp
edged constrictions, and carried out a preliminary testing
program in a small flume. The present study reports on the
design and construction of the flume for the main testing
program. The design was begun in July, 1958 and construction
was started in March, 1959. The flume and its appurtenances
were constructed and operative in August, 1959. Also in-
cluded is the testing of two dimensional semi-circular con-
strictions carried out from September 1959 to November 1959.

1.   Superscripts refer to bibliography at end of text.

REVIEW OF THE LITERATURE

Lane[3], in 1915 and 1916, performed several exper-
iments on contracted openings of various types. The full
series of tests was not completed but data was collected
and evaluated on the following:

1. Sharp edged vertical contractions

2. A rounded edge vertical contraction

3. A short flume with rounded entrance

4. A short flume with sharp corner entrance

5. An expanding flume

The paper describes the apparatus and procedure util-
ized as well as the results obtained. Lane found that the
Weisbach formula ($Q = C_D \, b \sqrt{2g} \left[\frac{2}{3} H^{3/2} + y \, H^{1/2} \right]$ *
where H is the total drop of the water in passing through
the contraction including the change in the velocity head
and y is the average downstream depth.) was most nearly in
agreement with the experimental results obtained in the case
of the sharp edged contractions.

Kindsvater and Carter[4] attempted to correlate labor-
atory results with field data obtained by the USGS. A gen-
eral equation was developed for discharge through a con-
tracted opening: $Q = C_D \, b \, y \sqrt{2g \left(\Delta h - h_f + \alpha \frac{V^2}{2g} \right)}$ where y

* See list of symbols in Appendix A

is the downstream depth. This expression contained a dis-
charge coefficient which was determined for several cases.
The base curves of the discharge coefficient in terms of
the contraction ratios and length of the constriction were
presented for a standard condition of square abutment and
Froude number of 0.5. The base curves are supplemented by
auxiliary curves modifying the coefficient due to:

    1.   Froude number different from standard

    2.   Upstream corner rounding

    3.   Eccentricity

    4.   Skew

    5.   Channel contraction

    6.   Chamfers of the abutments

    7.   Side depths

    8.   Abutment side slopes

    9.   Submergence

    10.   Bridge piers and piles

Tracy and Carter[5] made a laboratory study in 1954 of
the flow through contractions. The data was used to develop
a set of base curves relating the backwater ratio ($h_1^*/ \Delta h$)
and the contraction ratio m for different values of roughness.
These base curves were obtained using vertical-faced con-
strictions with square edged abutments, and a Froude number
of 0.5. Auxiliary graphs were presented to modify the result
according to the geometry of the constriction if other than
the basic geometry.

Vallentine[6] used vertical sharp edged constriction
plates placed normal to the flow. The discharge through the

constriction was related to the upstream depth by an equation
containing an experimental discharge coefficient: $Q =$
$C_D \, b \sqrt{2g} \; y_1^{3/2}$. The variation of this discharge co-
efficient with the Froude number of the unconstricted flow
was given for several values of the contraction ratio.

Nagler[7] did considerable experimental work with three
dimensional models of bridge piers. Although he worked at
an extremely small scale, several curves are presented re-
lating the backwater to the geometry of the pier.

Husain[1] carried out two and three dimensional tests of
arched openings on a small scale in preparation for a larger
study of which the work reported here is a part. The prob-
lem was also approached by dimensional analysis. General
profiles of the backwater curves were obtained and recom-
mendations were made concerning the design of the future
equipment.

Sooky[2] carried on the small scale testing program be-
gun by Husain and included two and three dimensional models
with and without channel roughness. For the two dimensional
case, equations relating the discharge to the geometry of
the opening and the backwater height were developed. Curves
were obtained from experimental data which relate the coef-
ficient of discharge and the backwater height to the Froude
number of the unconstricted flow and the contraction ratio.
These equations are presented in Appendix B.

DESIGN AND CONSTRUCTION OF THE APPARATUS

Testing Flume

Preliminary Computations

Before the design of the flume itself could proceed, it was necessary to determine whether the backwater and regain phenomena could be represented to a convenient and easily measurable scale in the space available in the hydraulics laboratory.

Several sets of arch bridge plans provided by the Indiana State Highway Department were analyzed for the values of backwater. The theory of varied flow and the equations and tables presented by Bakhmeteff[8] and by the U.S. Bureau of Public Roads[9] were used. It was only possible to make an approximate calculation of the backwater because of the unknown effect of an arch-type constriction. A sample calculation is presented for bridge S79 (Clay County, Indiana).

Assumption

1) Velocity through the bridge constriction
= 6 feet/second. This value was suggested
by Mr. J. I. Perry, Chief Engineer, Indiana
State Flood Control and Water Resources
Commission, as an average flood flow vel-
ocity through typical bridges in Indiana.

From bridge plan

Waterway area through the bridge utilized by the
stream = 125 feet^2

Slope of the stream bed = 0.0019 feet/foot

Flow = 750 cfs

The total waterway area under the arched constriction
of the bridge was approximated by a trapezoid. (See figure
1 appendix C) The unconstricted area of the waterway up-
stream of the bridge was found to be 300 feet^2.

In order to find the percentage of constriction of the
stream it was necessary to proportion the flow within the
subsections of the original area.

The tabulation for the proportionment is shown below.

Sub Section	n	1.486/n	A feet^2	p feet	R feet
1	.05	29.8	45	12	3.75
2	.05	29.8	210	30	7
3	.05	29.8	45	12	3.75
			A = 300		

Sub Section	$R^{2/3}$	$AR^{2/3}$	K cfs	Q cfs
1	2.41	108.5	3,240	86
2	3.65	776.0	22,800	578
3	2.41	108.5	3,240	86
			K = 28,280	Q = 750

The proportionment was made on the basis of conveyance.
The interface of sub sections was considered as a boundary.

The total waterway area through the bridge was 143.5 feet^2 located entirely in sub section 2. The loss in capacity based on relative conveyance was computed as follows.

Area deducted section 2 = 66.5 feet^2

(see figure 1 appendix C)

Area total section   2 = 210.5 feet^2

Flow deducted section 2 =

(66.5/210) x 578   = 182   cfs

Flow deducted section 1 = 86   cfs

Flow deducted section 3 = 86   cfs

Total flow deducted   = 354   cfs

The loss in capacity represented by this constriction is 354 cfs. The contraction ratio M then is (354/750) x 100 or 47%.

The following computations are based on data taken from the U.S. Bureau of Public Roads report.[9]

Assuming the geometry of a 30° wing wall normal crossing, the backwater coefficient K_b = 1.075 for a contraction. ratio M = 47%

$$V^2/2g = 36/64.4 = .56 \text{ feet}$$

$h_1{}^*$ = 1.075 (.56) = 0.602 feet. This is the maximum back-water superelevation.

Length of Backwater Profile. The backwater profile was computed using Bakhmeteff's[8] function $\phi(\eta)$. The computations made on this basis neglect the regain of kinetic energy in an expanding stream. In order to use the tables of $\phi(\eta)$, the hydraulic exponent n characteristic of the channel upstream of the bridge was computed by use of the

equation n = 2 cot $\propto$. ($\propto$ is shown in figure 2 appendix C)

y = 3 feet $\qquad\qquad$ y_1 = 7 feet

A = 90 + 18 = 108 feet^2 $\qquad$ A_1 = 300 feet^2

p = 30 + 12 = 42 feet^2 $\qquad$ p_1 = 44 feet

R = 108/42 = 2.57 feet $\qquad$ R_1 = 300/44 = 6.82 feet

C = 1.486/.05 (2.57)$^{1/6}$ = 34.8 $\quad$ C_1 = 1.486/.05 x (682)$^{1/6}$ = 41.0

K = 108 x 34.8 x 1.6 = 6000 $\qquad$ K_1 = 300 x 41 x 2.61 = 32,100

Log K = 3.78 $\qquad\qquad$ Log K_1 = 4.506

Log y = 0.478 $\qquad\qquad$ Log y_1 = 0.846

$\qquad$ n = 2(4.506 - 3.78/0.486 - .478) = 2(.726/.368) = 3.95

y	η	ϕ	$\Delta\phi$	l	L
feet				feet	feet
7.602	1.086	0.660			
7.500	1.070	0.599	.061	628	628
7.400	1.058	0.544	.055	565	1193
7.300	1.041	0.446	.098	1010	2203
7.200	1.030	0.362	.084	865	3068
7.100	1.013	0.137	.255	2320	5388
7.050	1.008	0.004	.133	1370	6758
7.020	1.003	0.264	.278	2860	9618

$\qquad$ Original normal depth = 7.00 feet

$\qquad$ Backwater $\qquad\qquad$ = 0.60 feet

$\qquad$ Depth at maximum

$\qquad\quad$ backwater $\qquad$ = 7.60 feet

$\qquad$ Superelevation

$\qquad\quad$ decrease $\qquad$ = 0.58 feet

This reach then includes 0.58/0.60 x 100 or 96% of the superelevation.

Complete information is lacking on the length of the regain curve.

It may be assumed that the angle of expansion of the stream downstream of the contraction may be approximated by the divergence angle of a submerged jet, and that the regain curve will be complete when the expansion has reached the full width of the flume. Albertson[10] found a divergence angle on each side of the centerline of a free jet of 11 to 14 degrees. Henry[11] found the free boundary to diverge at approximately 7 degrees for the flow from a submerged sluice gate. An angle of 5° is estimated with this particular bridge in order to obtain an approximation by excess of the regain curve as follows. (See figure 3, appendix C).

Clear span of bridge at spring line = 30 feet.
Surface width of arch at maximum high water = 11 feet.
Average width of opening = (30 + 11)/2 = 20.5 feet.
Stream width = 46 feet

The regain to normal depth should therefore be complete in a length of 146 feet measured from the downstream side of the constriction. This approximate computation indicates that the total length of the backwater curve reach (within 0.02 feet of the normal depth) and the estimated regain curve plus the bridge is 9618 + 146 + 48 = 9812 feet. It is now desired to find the required length of flume to represent to scale the totality of the regain curve and a

reasonable portion of the backwater curve.

The water flow available in the hydraulic laboratory was 2100 Gpm.

2100 Gpm = 2100/449 = 4.7 cfs.

Since $Qm/Qp = L_r^{5/2}$ the required scale ratio is found to be $4.7/750 = L_r^{5/2} = 0.00627$.

$$Lr = (.00627)^{2/5} = .135$$

or $Lr = 1:7.4$

The nearest convenient scale is 1:10. For this ratio, the superelevation = .06 feet = .72 inches. The usable length of the flume in the proposed space would equal 60 feet. For this ratio, 600 feet of the prototype stream could be represented. This would include the totality of the regain curve, the bridge model and some 400 feet of the prototype backwater curve.

From this and similar computations as well as small scale tests,[1] it appeared that a flume utilizing all of the easily available space in the laboratory that is a flume length of 64 feet but capable of extension would be satisfactory. The width of the flume was fixed at 5 feet. This was based on a consideration of the scale ratios and the space available.

The cross section of the flume was to be rectangular since this configuration lent itself well to both ease of construction and adaption.

<div align="center">Design Procedure</div>

In order to test the flows under varying slope conditions, the flume was to be tiltable about one end.

Several methods are available to achieve slope control
including:

1.   Hydraulic Jacks

2.   Screw Jacks

3.   Wedges

4.   Cables and Winches

Screw Jacks were selected because of accuracy and ease
of control, as well as permanence and appearance.

At the time the preliminary design was made, only an
estimate could be made of the final weight of the flume and
the water contained therein.

In order to keep the deflections due to the variable
weight of water within the same order of magnitude of the
smallest readings of the point gage for depth measurement,
0.1 mm, the flume bottom was designed of 1/4 inch steel plate
supported at 2 foot intervals on channels. The channels in
turn were to be supported by two or more main beams riding
on the jacks. The side plates were designed of 1/4 inch
steel plate supported by vertical angles resting on the
channel members. A longitudinal horizontal angle mounted on
the vertical angles served as a support for the guide rails.
The guide rails, which serve as a reference plane from which
measurements are based, were to be polished stainless steel
to minimize corrosion and scale.

The preliminary design was based on a possible water
depth of 2. feet. In the immediately proposed tests this
will allow a freeboard of approximately 1 foot. However, de-
flection will be within the set limits for a loading of

2 feet of water which may occur at a later date.

A portion of the first design is presented.

Dead Load

1/4 inch plate

2 side plates 64 feet long x 2 feet wide

@ 20.4 lb/foot   = 2620 lb.

1 bottom plate 64 feet long x 5 feet wide

@ 51.0 lb/foot   = 3260 lb.

                                                    5880 lb.

Main Beam (18 I 54.7)

2 x 64 feet x 55 lb/feet =   7040 lbs.

Channels

33 x 5 feet x 8.2 lb/feet =   1360 lbs.

                                       14,280 lbs.

Extras @ 30%   4,300 lbs.

                          total   18,580 lbs.

For design purposes this is an average load of

18,580 lbs/64 feet = 290 lbs/foot.

Live Load. At the maximum depth of 2 feet the volume
of water contained is 10 feet^3 per foot of length. This
imposes a load of 10 feet^3 x 62.4 lbs/foot^3 = 624 lbs/foot.
The total load per foot then is 290 pounds/foot + 624 lbs/foot
= 914 lbs/foot.

The distance between supports was set as 20 feet. Since
the exact nature of the main beam connections was unknown, the
solution was made based on a simply supported condition.

Two alternatives were presented. The first was to use
beams whose stiffness would make any deflections negligible

and the second was to use lighter beams and correct for the
deflections by adjusting screws. The first alternative was
chosen as the second would necessitate adjustment after each
change in conditions such as slope, or water depth.

The beam first selected was an 18 I 54.7 which gave a
calculated deflection of 0.00225 feet under the design loading.
Contacts with the fabricator and erector were made at a later
date and it was found that a 20 I 65.4 would be available
at a cost less than that of the lighter beam. The use of
the heavier beam was accepted and the design proceeded based
on this beam. The deflection due to the variable water
weight was approximately 0.002 feet for a depth of one foot.

It was felt that some form of transverse leveling was
necessary. Adjustment bolts were an obvious solution but
the location was yet to be selected. The first sketches in-
corporated adjustment bolts between the channel and the bot-
tom plate. This produced an indeterminate situation with
regard to flexure, inconvenient locations for adjustment, and
high fabrication cost. The subsequent designs placed the
adjustment bolts between the channels and the main beams
which gave the bottom plate full support across its width at
2 foot intervals.

The bottom plate was designed slightly wider than the
flume width. This permitted attaching an angle to hold the
bottom edge of the vertical plate fixed. The upper edge
of the vertical plate had nuts welded on at the two foot
points. Studs were attached between the nuts and the

vertical angles to support the plate and provide an ad-
justment for its longitudinal alignment. The inside of the
flume was finished with an epoxy resin applied with a hand
roller. The flume construction is shown in figures 4 and 5.
(Appendix C)

In operations of this kind, guide rails are generally
fixed in the level position. They then may be used to con-
trol the slope. Since the rails were attached to the flume,
direct slope measurements was not possible. Instead, dif-
ferences between the surface of a standing body of water and
the flume floor were used to measure the slope and calibrate
a revolution indicator which served as the primary method of
slope control.

Tailgate

Control over the depth was exercised by a gate mounted
at the end of the flume. The gate was manually operated
from the side of the flume. Figure 6 (appendix C) shows
the gate. The gate was made in such a way that it could
be used either as a sluice or as a weir. Throughout this
first series of tests, the gate was used exclusively as a
weir.

Forebay

The forebay, 8 feet wide and 10 feet long, was con-
structed of plywood and lined with sheet metal, and is shown
in figure 10 of appendix C. The 3 inch and 6 inch pipes
entered the rear of the forebay at the top. The 6 inch

line was centered and the 3 inch line was placed slightly off center. The diffusing mechanism for each supply line consisted of a tee and cross pipe of the same size as the line at the bottom of the forebay. The turbulence of the entering water was controlled by a 4 inch gravel baffle and three wire mesh screens of 13 mesh per inch. The transition section continuing into the bottom and side walls of the flume was made of quarter ellipses in the horizontal and vertical planes respectively with a ratio of major to minor axes of 1.5 to 1.0. The joint between the flume and the forebay was sealed with a flexible rubber gasket mounted so as not to interfere with the flow.

Slope Control Mechanism

The flume rests upon six screw jacks and a hinge. The hinge is located at the joint of the flume and the forebay. The jacks are similar in all respects with the exception of the gear ratio. The jacks are divided into three pairs with rates of raise of one, two, and three inches for 96 turns of the shaft. Since the hinge was a fixed point and the opposite end of the flume was the point of maximum movement, the jacks were arranged such that the pair nearest the hinge moved the least and the pair at the opposite end of the flume had largest displacement per revolution. This maintained the bottom of the flume as a plane while it was being raised and lowered.

The jacks on each side of the flume were driven by a common 1 inch shaft line connected at one end to a 90° miter gear. The miter gears on either side in turn were connected

to a single 60:1 ratio gear reducer. The power to operate all the jacks was supplied by a 1-1/2 horsepower, 1750 revolutions per minute, reversible, electric motor connected directly to the gear reducer. This provides a rate of vertical displacement at the downstream jack station of approximately 1 inch per minute.

The jacks were arranged in such a way that the downstream end of the flume may move from 12 inches below horizontal to 3 inches above horizontal, resulting in a maximum positive slope of 1/60 and a maximum adverse slope of 1/240. The motor was controlled by a raise, lower, and stop control switch. Safety switches were located both near the motor and near the control switch. It was necessary to unlock these before the control circuit could be completed. In addition, automatic limit switches were provided to prevent running the jacks beyond their limits. The general arrangement of jacks and gears is shown in figure 7.

In order to connect the ends of the jacks (which move in a vertical line) to the flume (which moves in an arc), it was necessary to use a pinned linkage. A photograph of the linkage is shown in figure 8. (Appendix C)

Due to the arc of the linkage, the relation between the rise of the flume and the revolutions turned by the jack shafts was not linear. Therefore, it was necessary to make a calibration of the slope rather than computing it.

At the time of erection, the jacks were leveled at .001 foot before the flume was erected. During the alignment procedure the jacks were raised or lowered individually as needed

to obtain a level base. The shaft couplings were then in-
stalled an no further individual movements were made.

Construction

Considerable time was spent in obtaining the requi-
sitions, bids from several contractors and actual super-
vision of the erection of the flume. The foundations,
jack piers, plumbing, and electrical controls were installed
by Purdue University Physical Plant. The structural parts
of the flume were built and assembled by a contractor,
the flume adjustments, construction of the instrument
carriage, installation of manometers and calibrations were
done by the Research Assistant and student labor when needed.

PREPARATION FOR TESTING

Slope Calibration

After the flume was aligned and leveled a slope cal-
ibration was made by visually counting the revolutions of
the slowest speed shaft and measuring the depth of a still
pool of water at two points 50 feet apart. A steel tape was
installed with station 0 at the upstream end and station
64 at the downstream end of the flume. The points chosen for
slope measurement were stations 6 and 56. These points had
previously given consistantly good results when measurements
were made of the distance between the rails and the flume
floor. The calibration of slope versus revolutions appears
in figure 9. (Appendix C) The apparent scatter of the points
toward the downstream end of the flume is due to the magnifi-
cation resulting from the logarithmic scale at that end.

In order to avoid the necessity of visually counting
shaft revolutions to keep track of the slope, a revolution
indicator was made and installed at jack station number 3.
The lowering of the flume activated the pointer which both
multiplied and reversed the motion. The tip of the pointer
rode on a lucite strip mounted below the motor controls. A
mark was scribed on the lucite strip at each revolution over
the range 0 through 40 as well as at the test slopes. On
the end of the shaft a circular lucite plate divided into ten

parts was mounted along with a fixed pointer. The slope desired was set by using the large indicator to the nearest revolution and setting the tenths of a revolution by using the small dial. This equipment was later replaced by a commercial revolution counter mounted at the same location. This counter read directly to a tenth of a revolution and the reading could be interpolated to one half of that. This device provides a slope control with an accuracy of $\pm$ 0.0000025 feet/feet. Figure 8 (Appendix C shows the counter)

Water Supply and Measurement System

The water available in the laboratory is recirculated through the system by two pumps rated at 300 Gpm and 2000 Gpm. The 3 inch discharge line from the 300 Gpm pump was connected to a new 3 inch line. This line contained a new 3 x 2.25 inch venturi accurate to 0.5% over the range from 30 Gpm to 300 Gpm. The line was installed using long sweep elbows to reduce head loss. A 60 inch differential manometer reading to 0.01 inch was connected to the venturi and filled with tetrabromoethane (Specific gravity 2.95) which gave a manometer deflection of 51 inches with a flow of 336 Gpm.

The 2000 Gpm pump was connected to an existing 6 inch line which was improved by the installation of long sweep elbows in place of tee's and short elbows. In addition, a new 6 x 4.176 inch venturi accurate to 0.5% over the range 200 Gpm to 2000 Gpm was installed preceeded by a set of straightening vanes. A 30 inch differential manometer reading

to 0.01 inch was connected to the venturi and filled with
mercury (Specific gravity 13.6). This manometer gave a de-
flection of 14.9 inches for a maximum flow of 1790 Gpm.
In both cases the venturis were fitted with air vents to
insure proper measurements. After a portion of the tests
had been made and the data evaluated, it was deemed necessary
to improve the discharge measurements. The 60 inch manometer
was attached to the 6 inch venturi and recalibrated using tet-
rabromoethane as the manometer fluid. This resulted in a
larger manometric deflection improving the accuracy of the
measurement. The 30 inch manometer was connected to the
3 inch venturi but there was no need to recalibrate the meter.

In order that the calibration of the venturi meters should
have no error larger than that of the venturi meter, the scale
to be used for the calibration was checked against standard
weights by the Indiana State Board of Health, Division of
Weights and Measures. The scale error was less than 0.2%
or 2 pounds per 1000 pounds. For the purpose of calibrating
of the venturi meters, branch lines led to a baffled concrete
channel located above the weighing tank.

At the point immediately before the 3 inch and 6 inch
lines entered the forebay, valves and valve bypasses were
installed. The 6 inch line had a 2 inch bypass and valve
and the 3 inch line was fitted with a 1 inch bypass and
valve. The manometers were mounted in a position easily
visible to the person adjusting the valves. The overall
apparatus arrangement in the laboratory is shown in figure
10. (Appendix C)

Venturi Calibration

As soon as the essential piping was completed, cal-
ibration of the venturi meters was begun. The procedure was
as follows. The flow was set using a valve located downstream
of the venturi and a waiting period of approximately 5 minutes
was allowed for the system to come to equilibrium under the
new flow. The scales were preset to an arbitrary weight and
the weighing tank valve closed. The manometer deflection was
noted and the water diverted into the weighing tank. The
scales were tripped and the timer manually started when the
weight of water collected equalled the weight which had been
preset on the scale beam. The scale weight was noted. This
method of calibration avoids the errors of non instantaneous
starting and stopping of flow but still does not correct or
make allowances for the difference in the impact force of the
water entering an empty tank as compared to a partially full
tank. This error is of the magnitude of 1% which is com-
patable with accuracy of the remainder of the system. The
intervals of calibration were selected so as to fall between
one and two inches of deflection on the 60 inch manometer
containing the lighter fluid and not to exceed 1 inch on the
30 inch manometer containing the mercury. The calibration
curves are presented in figures 11, 12 and 13. (Appendix C)

MODEL TESTING

Model Construction

From results of the preliminary experiments,[2] (figure
14 Appendix C) it was found that the predominant variable
was the contraction ratio and that the length of the model
had little or no influence for Froude numbers less than 0.5.
This range of Froude numbers corresponds to the case usually
found in practice. It was therefore decided that the first
series of tests would concentrate on two dimensional sharp
edge semi-circular models with no skew.

The cost of machining mild steel plates to produce the
constrictions was prohibitive. An estimate was obtained of
$125.00 per model. The frequency of handling and changing
the constriction plates also necessitated a model that was
light weight and still capable of sustaining hard use without
damage. The models, as finally made, consisted of back up
sheets of 1/2 inch exterior grade plywood faced with a sheet
of 22 gauge galvanized iron and braced by a steel angle. The
openings were cut out of the galvanized iron sheet accurate
to 1/32 inch. The opening in the plywood backing had a radius
of 1/2 inch greater than that used in the metal. The metal
was bolted tightly against the wood. In the flume, the model
was positioned with the metal face upstream. The construction
of the models is shown in figure 15. (Appendix C)

Four models were made with contraction ratios of 0.3,
0.5, 0.7, and 0.9. At the slopes and flows tested, the
model with m = 0.3 was submerged on a majority of the
tests. The data presented in Appendix C is that collected
for the other three models which are shown in figure 5.
(Appendix C)

Free Surface Measurement

The position of the free surface was measured with an
electric indicating point gage reading to the nearest 0.1 mm.
The point gage was mounted on an aluminum and brass bar in
such a way that the gage could traverse the width of the flume.
This bar in turn was part of a carriage which rolled on the
stainless steel rails along the flume. The carriage was
rectangular and rolled on 4 wheels, two of which on one side
of the carriage were grooved to provide alignment. The
power supply was mounted at the back of the carriage, The
operator rode on a second carriage which was on its own rails.
Details concerning the instrument carriage can be seen in
figure 16. (Appendix C)

The head of the point gage was com prised of two probes
approximately 1/4 inch in diameter. The probes were sep-
arated a distance of about 1-1/2 inches. The rear electrode
was 15 millimeters longer than the front, and served as the
ground. The end of the front electrode was pointed and was
adjusted to the water surface. In operation, the obstruction
to the flow presented by the rear electrode caused a rise in
the water surface of $v^2/2g$ against the electrode. The effect
of this disturbance extended upstream to the measuring probe

and made it impossible to measure the true water surface. This effect was not a constant since the velocity head was different in each case.

In order to improve the performance of the instrument, the rear electrode was replaced by a small diameter copper wire. This wire presented a much smaller obstruction to the flow and the operation of the gage was not only simplified, but gave data that should be superior to that obtained with the former arrangement.

Uniform Flow Calibration

Preliminary tests were run to determine uniform flow conditions in the flume. The variables involved are: the discharge, the slope and the tailgate settings. For each flow and slope the tailgate setting was determined by trial and error until uniform depth was obtained along the largest pos- sible reach of the flume. The models were located in such a way that the regain curve was complete within the uniform depth section. Effects of boundary layer growth were not considered, and the flow was considered uniform as long as the depth remained constant.

As the model tests progressed the uniform flow con- ditions were recorded in the auxiliary graphs of figures 20 and 21. (Appendix C)

The first was the normal depth versus the slope and the second was the height of the tailgate versus the slope. In both cases, the rate of flow was the parameter. In order to obtain data that would provide a complete coverage of the

range of Froude numbers investigated, the Froude number was
first chosen and the normal depth computed in the following
manner.

$$F_o = \frac{V}{\sqrt{g y_o}} = \frac{Q}{y_o \, B \, \sqrt{g y_o}}$$

or

$$y_o^3 = \frac{Q^2}{F_o^2 \, B \, \sqrt{g}}$$

from which

$$y_o = \left(\frac{Q^2}{F_o^2 \, B \, \sqrt{g}} \right)^{1/3}$$

After the desired normal depth was computed, the slope
for a particular flow which would give this value could be
determined from figure 20 (Appendix C). Then, entering fig-
ure 21 (Appendix C) with the slope and flow, the necessary
gate setting to produce uniform flow could be determined.

Testing

After the slope and uniform flow calibrations were com-
pleted the testing was begun. The procedure used was to set
the flow, which was the most difficult quantity to adjust, and
let it remain constant while the slope, tail gate setting
and models were changed. For each case of slope and flow,
the tail gate heighth was set according to the previous un-
iform flow calibrations. In the majority of cases the con-
ditions of uniform flow was obtained on the first trial and
in every case no more than two trial gate settings were
needed. Once the system had come to equilibrium and the

normal depth was obtained, a model was installed in the
flume. For the first few runs, the complete profile was
taken. It was observed that in the latter portion of the
regain curve, the water surface fluctuated vertically as
much as one centimeter from one minute to the next. This
fluctuation did not occur rapidly but rather slowly. Testing
was immediately suspended while the cause for this was deter-
mined. The first possible cause investigated was that of a
variable inflow into the flume. However, observation of the
backwater showed it to be very stable. This indicated that
the flow into the flume was constant and the phenomena was
due to the model or that portion of the flume beyond the model.
Eddies were suspected as a possible cause of this phenomenon.
In an effort to eliminate eddies, the gap between the metal
facing and the plywood back up board was filled and beveled
to produce an angle of 45°. This resulted in no appreciable
change. The condition most suspect however, was channelization
of the flow to one side of the flume with eddies on the opposite
side. It seemed that an instability was developed by the
constriction. The reason for this channelization of the flow
is not known. Misalignment of the bottom was not a factor
because it was level within 1 millimeter throughout the length
of the flume. The tail gate was leveled to an accuracy of
1 millimeter and the models were installed, using a square
to check the alignment both horizontally and vertically. It
was discovered that in a given case with most of the flow on

one side, after artificially deflecting the flow to the other
side, it would remain on that side. This suggested that the
lack of symmetry was not a factor. One possible solution is
the addition of roughness and increasing of slope to sta-
bilize the flow. However, since these tests were to be
specifically smooth boundary tests, this was not done and
the affected portion of the regain section was neglected. For
the remainder of the tests, only a short profile before and
after the model was taken to locate the points of maximum
and minimum depth. Figures 17, 18, and 19 (Appendix C)
are photographs of the flume with model in place. Figure
18 shows the flow going to one side and figure 19 shows the
flow centered.

Test Results

The test data and the calculated values of the dis-
charge coefficient C_D, of the friction coefficient f, and
of the Reynolds number are presented in table I. The ratio
of the backwater depth to the normal depth y_1/y_o is plotted
versus the ratio of the velocity head to normal depth with
the contraction ratio m as a parameter in figure 22. The
ratio of the velocity head to the normal depth is a measure
of the kineticity of the flow, it is exactly half of the kin-
eticity as defined by Bakhmeteff[3], or half the square of
the Froude number. The discharge coefficient C_D calculated
from equation 2 of Appendix B is plotted versus the ratio of
the velocity head to the normal depth with the contraction
ratio m as a parameter in figure 23.

The consistency of the data is well illustrated by the lack of scatter of the experimental points as plotted in figures 22 and 23. (Appendix C) These test results may be compared with the small scale tests for the contraction ratio of 0.5 which is common to both test series. Inspection of figure 14 (Appendix C) and figures 22 and 23 (Appendix C) show that the values are almost identical. For example, at a Froude number of 0.2, the value of the discharge coefficient C_D from the small scale tests (figure 14 Appendix C) is 0.38 and the superelevation ratio y_1/y_o is 1.1. Compared to this, the large scale tests, (Figure 22 and 23 Appendix C) give C_D as 0.275 and y_1/y_o as 1.119 for a kineticity of 0.02 which corresponds to the Froude number of 0.2. At a Froude number of 0.4 the results of the small scale tests indicated C_D was equal to 0.53 and y_1/y_o was 1.4. Similarly, at a corresponding kineticy of 0.08, the large scale tests showed C_D to be 0.483 and y_1/y_o to be 1.432. The reliability of the data may be better discussed in terms of the values of the friction coefficient and of the backwater ratio.

The Darcy-Weisbach friction factor and the Reynolds number for the uniform flow established before each test were calculated in table I as follows:

$$R = \frac{VR}{\nu}$$

$$f = \frac{8gRS}{V^2}$$

The experimental friction factors were compared to the theoretical values obtained by adapting the Blasius and Prandtl formulas for flow in smooth pipes to the rectangular open channel.

The formulas for smooth pipe flow are:

Blasius $f = 0.3164 \left(\frac{VD}{\nu}\right)^{-1/4}$ $R < 10^5$

Prandtl-Von Karman $\frac{1}{\sqrt{f}} = 2.0 \log \left(\frac{VD}{\nu} \sqrt{f}\right) - 0.8$

Replacing D by 4R and simplifying, the equations become:

Blasius $f = 0.223 \left(\frac{\nu}{VR}\right)^{1/4}$

Prandtl-Von Karman $\frac{1}{\sqrt{f}} = 2.0 \log \left(\frac{VR}{\nu} \sqrt{f}\right) + 0.40$

Powell and Posey[12], working with a triangular flume found the formula governing their friction factor to be

$$\frac{1}{\sqrt{f}} = 2.074 \log \left(R \sqrt{f}\right) - 0.797$$ for

tranquil flow in a smooth channel. The comparison between experimental values and the theoretical formulas is shown in figures 24 and 25. (Appendix C) In figure 24 the experimental values of the Darcy-Weisbach coefficient f are plotted versus the Reynolds number. Both f and R are as defined above based on the hydraulic radius. Also plotted on the same figure are the Blasius and Von-Karman relationship as well as the general range of experimental values obtained by Lansford and Robertson[13] for smooth triangular channels.

In figure 25 (Appendix C) the friction coefficient f and the Reynolds number are based on the normal depth, assuming two dimensional flow. Although this assumption is not completely true for the depth to width ratio used in

the experiments this was done to compare the data with the
experiments of Owen[14] which were done in a glass channel.

In general, as shown in figure 24, (Appendix C) the data
fall above the theoretical lines which are a lower limit
for a perfectly smooth boundary. The average friction co-
efficient is f =0.021 which corresponds to an absolute
roughness of approximately $\mathcal{E}$ = f (f) = 0.0025 feet. This
corresponds to the irregularities of the epoxy resin of the
channel finish.

The percentage of probable error of the friction co-
efficient is calculated as follows. The calculations are
made based on a flow of 3 cfs and a slope of 0.000100 foot/feet.
The measurement of the flow during calibration had a possible
error of 1% due to neglecting the impact in the weighing
tank with different water depths, and the scales had a pos-
sible error of 0.2%. The possible error in the venturis
was 0.5% and the observed manometer readings could have
been in error by 0.02 inches. With the given flow, the
manometer deflection was 52 inches of tetrabromoethane.
This means that the possible error in reading the deflection
value was approximately 0.02/54 or 3.85%. Since the flow
varies with the 0.5 power of the deflection, this would
represent an error in the flow of 1.93%. Therefore, the
total possible error in the flow is 3.63%. The slope was
calibrated and set to less than one tenth revolution of the
connecting shafts. A maximum error of one twentieth of a
revolution is equal to 0.0000025 feet/feet. At the slope

chosen, this would produce an error of 0.0000025/.0001 or
2.5%. The measurement of y_o was made to 0.1 millimeter.
An error of this magnitude with the value of y_o found for
this condition (21.70 cm) amounts to 0.01 cm/21.70 cm = 0.046%.
The error in computing the wetted perimeter could be
2(0.01 + 0.5)cm/192 cm = 0.053%. Since $f = 8gRS/v^2$ or
$8gSy_o^3/Q^2p$, the error in f can be expressed as

$$\sqrt{2.5^2 + (3 \times 0.046)^2 + (2 \times 3.63)^2 + (0.053)^2} = \sqrt{58.97} =$$

7.68.

At flatter slopes or lower flows, this error would be
even larger than calculated. Conversly, those tests at
steeper slopes and higher flows should give the most nearly
correct values of f. However, the points would not be ex-
pected to fall on the theoretical line since the materials
used in the construction of the flume and the finish applied
certainly caused some finite value of roughness.

A second check was made by determining the backwater
ratio $h_1^*/\Delta h$ for each model test and comparing the values
obtained to those presented by Tracy and Carter[6] for
rectangular constrictions. This comparison is shown in
figure 26 (Appendix C) which shows that the backwater ratios
are of the same order of magnitude although different as
may be expected with different boundary geometries.

SUMMARY OF RESULTS AND CONCLUSIONS

Flume Design

The flume, as designed and built has proved adequate
to carry out the proposed experiments. The entire apparatus
necessary to carry out the testing program can be run con-
veniently by two men. The slope controls, including the
jacks and motor, function well and changing slope takes less
than 5 minutes.

Testing

Only the first series of tests, sharp edged, semi-
circular constrictions, have been completed on a large scale
so comparisons cannot be drawn as such. However, several
things indicate the validity of the data.

1. Close agreement with the small scale tests.

2. Comparison with other investigators on:

a. Friction factor.

b. Mannings roughness

3. Adherance of the plotted data to a well
defined pattern with little experimental
scatter.

The conclusions possible so far are primarilly drawn
from figure 23 which relates the discharge coefficient and
the kineticity of the flow, and figure22 which shows the
backwater in terms of kineticity. From the first, it can

be seen that above a value that corresponds with a Froude
number of 0.5, the C_D value ceases to depend on the kin-
eticity of the flow. The second, figure 22 shows the de-
pendance of the backwater value on both the contraction ratio
and the kineticity.

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APPENDIX A

NOTATIONS

NOTATIONS

SYMBOL	UNIT	DEFINITION
A	feet^2	Area of flow.
B	feet	Stream width at bridge site or flume width.
b	feet	Width of constriction opening equal to diameter of semi-circle.
C_D		Coefficient of discharge equation.
D	feet	Pipe diameter.
E		An infinite series of powers of y_o/r.
F		Froude number of flow. Subscript o refers to unconstricted stream.
f		Darcy-Weisbach friction factor.
g	feet/sec^2	Acceleration of gravity.
h_1^*	feet	Superelevation of backwater above normal depth.
h_f	feet	Boundary friction loss.
Δh	feet or centimeters	Elevation difference between points of maximum

		and minimum depth.
K_b		Backwater head loss co-efficient.
K	cfs	Conveyance of a channel or section of channel $= AC \sqrt{R}$.
l	feet	Length of reach in back-water computations.
L	feet	Accumulated length in backwater computation.
Lr		Scale of length.
M		Channel contraction ratio equal to 1-b/B (in review of lit-erature).
m		Channel contraction ratio equal to b/B.
n	feet	Manning roughness or hydraulic exponent in $K^2 = $ const. y^n .
p	feet	Wetted perimeter of stream or subsection.
Q	cfs	Total flow (subscripts m and p refer to model and prototype).
q	cfs	flow in a subsection (subscripts m and p

		refer to small model flume and to prototype flume.)
R	feet	Hydraulic radius.
r	feet	Radius of semi-circular constriction.
$I\!R$		Reynolds number.
S	feet/feet	Slope of stream bed or flume.
V	feet/sec	Average velocity (subscript o refers to unconstricted flow).
y	feet or centimeters	Depth of flow.
y_o	feet or centimeters	Normal depth of flow in unconstricted channel.
y_1	feet or centimeters	Maximum depth of flow upstream of constriction.
α		Slope of line on Conveyance versus Depth graph.
β		Ratio of bottom slope to critical slope.
η		y/y_o
ϕ		Bakhmeteff backwater function.

APPENDIX B

EQUATIONS OF FLOW

DERIVATIONS OF EQUATIONS GOVERNING THE FLOW IN RECTANGULAR

CHANNELS WITH SEMI-CIRCULAR CONSTRICTIONS

The equation for the discharge in rectangular channels with a sharp crested semi-circular constriction is obtained and is expressed in terms of an infinite series of powers of the ratio y_1/r. With reference to Figure 27, (Appendix C) Bernoulli theorem gives

$$V = C\sqrt{2gh'} = C\sqrt{2g(y_1 - h)}$$

The element of area is

$$dA = 2\sqrt{r^2 - h^2}\, dh$$

and the discharge is thus

$$Q = \int V\,dA = \int_0^{y_1} C\sqrt{2g(y_1-h)} \cdot 2\sqrt{r^2 - h^2}\, dh \quad (1)$$

Expanding into a series, integrating term by term, and making use of the fact that $2r = b$:

$$Q = C_D\sqrt{2g}\,\frac{17}{24}\,y_1^{3/2}\,b\left[1 - 0.1294\left(\tfrac{y_1}{r}\right)^2 - 0.0177\left(\tfrac{y_1}{r}\right)^4 + \cdots\right] \quad (2)$$

This may be written as

$$Q = C\,y_1^{3/2}\,b\,E \quad (3)$$

where

$$C = C_D\,\frac{17}{24}\sqrt{2g} \quad (4)$$

and

$$E = \left[1 - 0.1294\left(\tfrac{y_1}{r}\right)^2 - 0.0177\left(\tfrac{y_1}{r}\right)^4 + \cdots\right] \quad (5)$$

The discharge in the rectangular flume is given by

$$Q = V_0 A_0 = F_0\sqrt{g}\,B\,y_0^{3/2} \quad (6)$$

where

$$F_0 = \frac{V_0}{\sqrt{g\,y_0}} \quad (7)$$

is the Froude number of the undisturbed flow. Equating
(2) and (6) and solving for the coefficient of discharge

$$C_D = \frac{12\sqrt{2}}{17} \cdot \frac{F_o}{mE} \left(\frac{y_o}{y_1}\right)^{3/2} \tag{8}$$
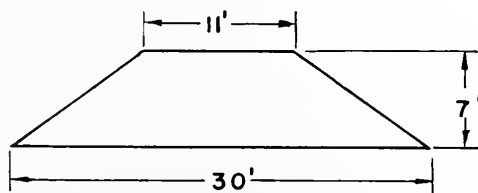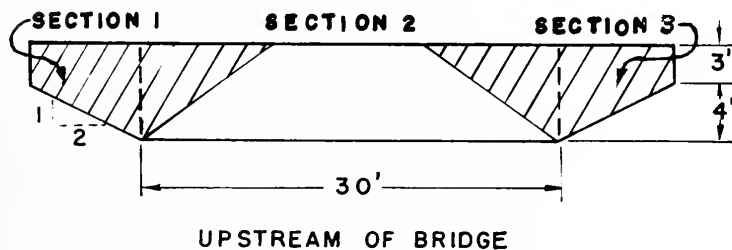
where

$$m = \frac{b}{B} \tag{9}$$

or

$$\frac{y}{y_o} = \left[\frac{12\sqrt{2}}{17} \cdot \frac{F_o}{mE\,C_D}\right]^{2/3}$$

APPENDIX C

FIGURES AND TABLES

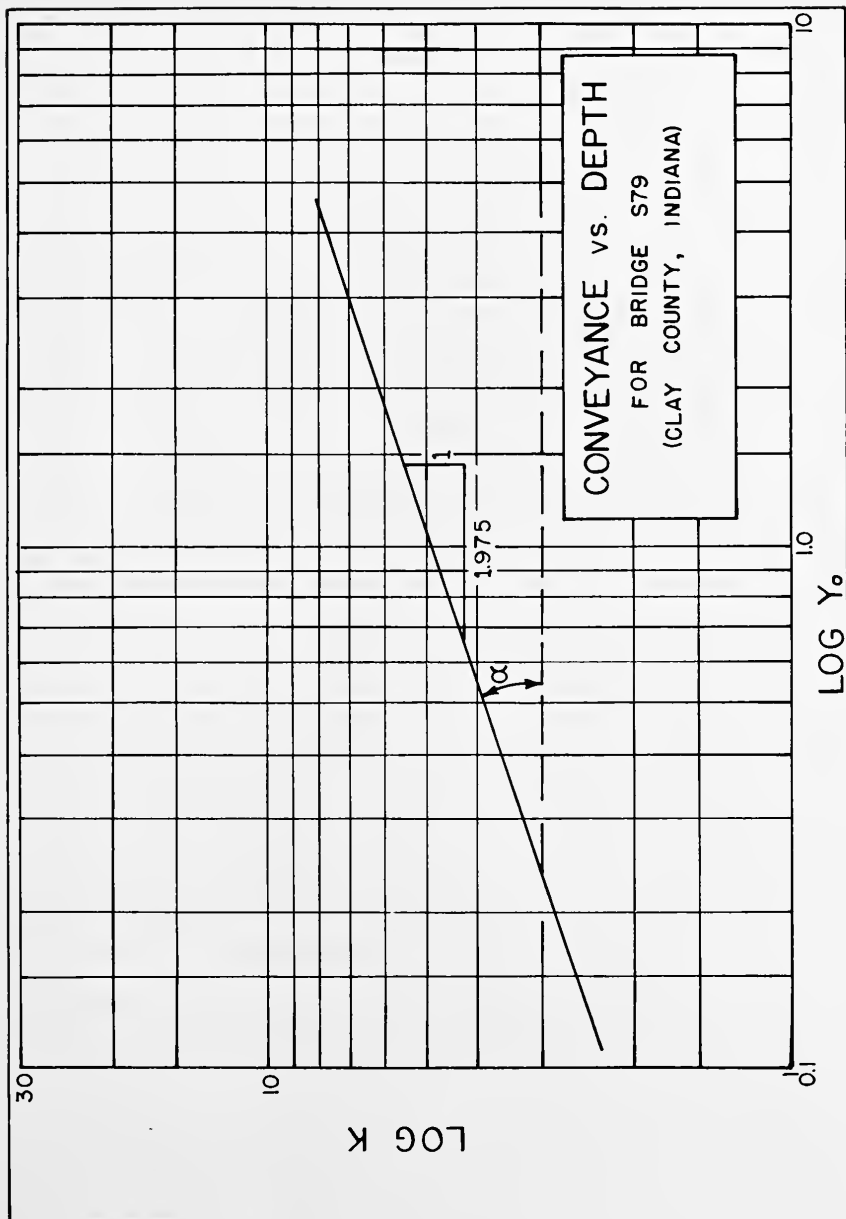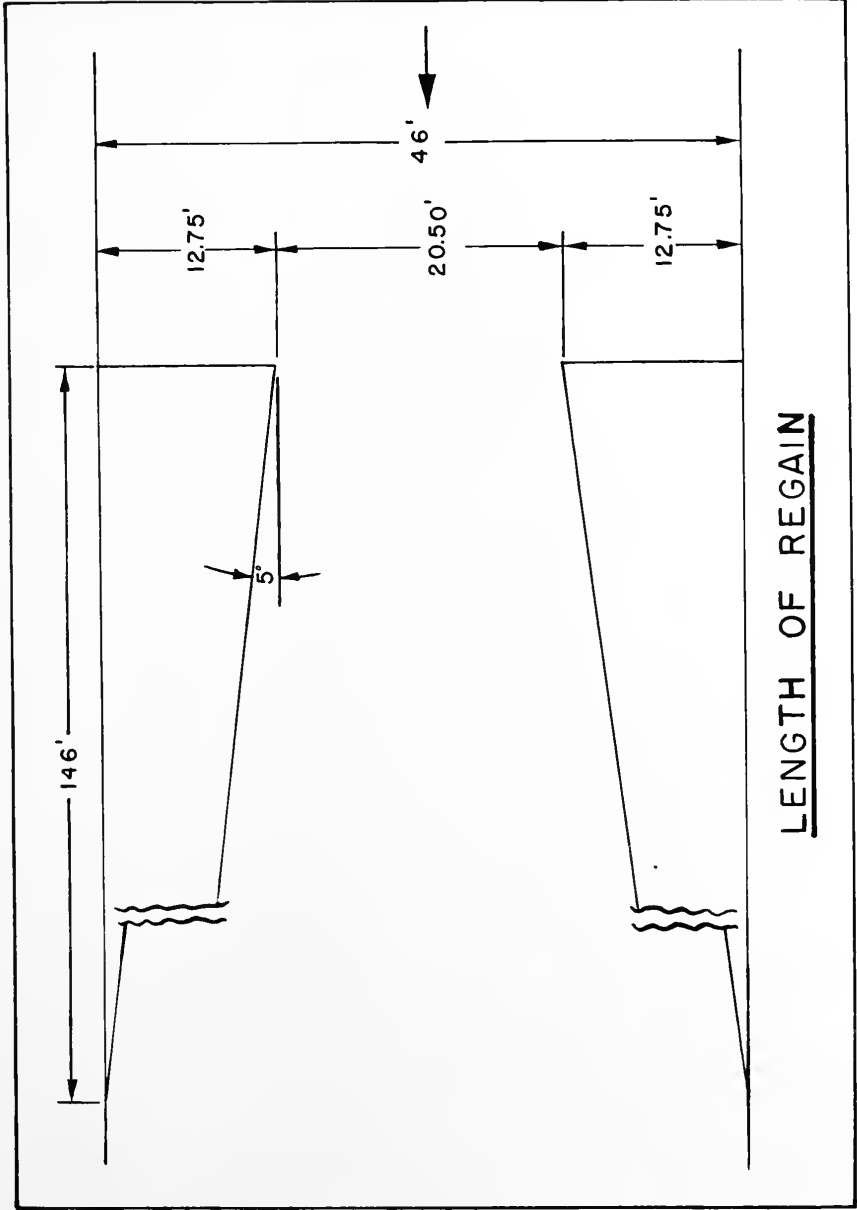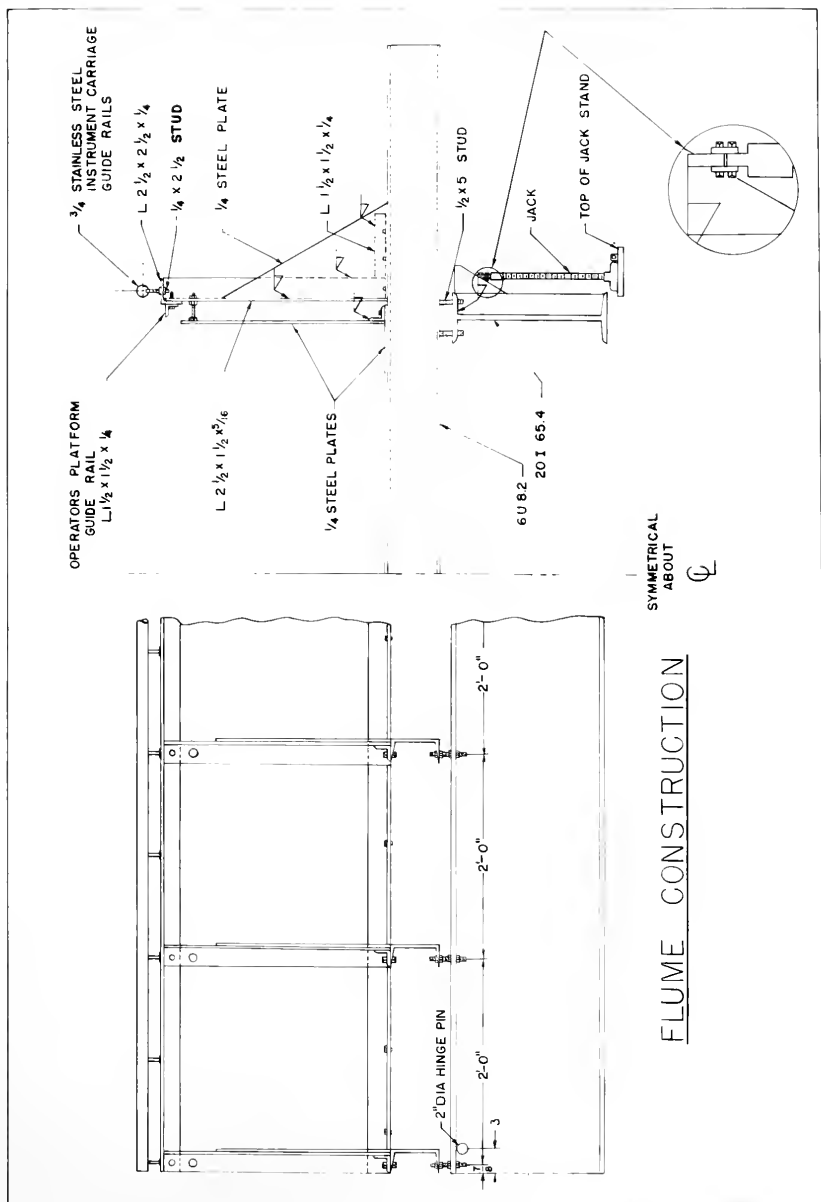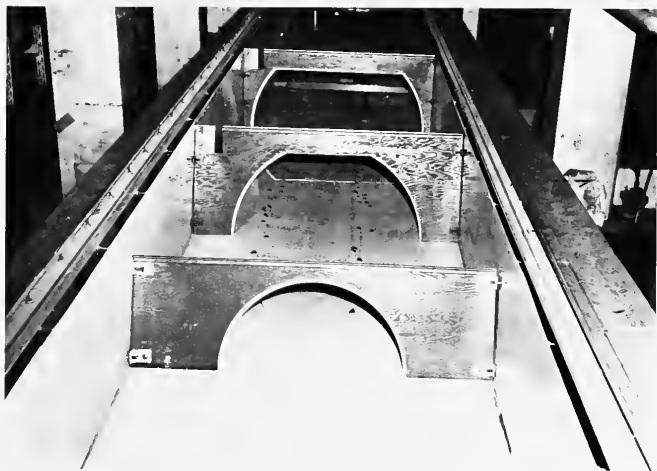IDEALIZED STREAM CROSS SECTION

FIGURE 1

CONVEYANCE vs. DEPTH
FOR BRIDGE S79
(CLAY COUNTY, INDIANA)

FIGURE 2

LENGTH OF REGAIN

FIGURE 3

FLUME CONSTRUCTION

FIGURE 4

FIGURE 5

FLUME AND MODELS

FIGURE 6

TAIL GATE CONSTRUCTION

PLAN VIEW OF JACKS AND GEARS
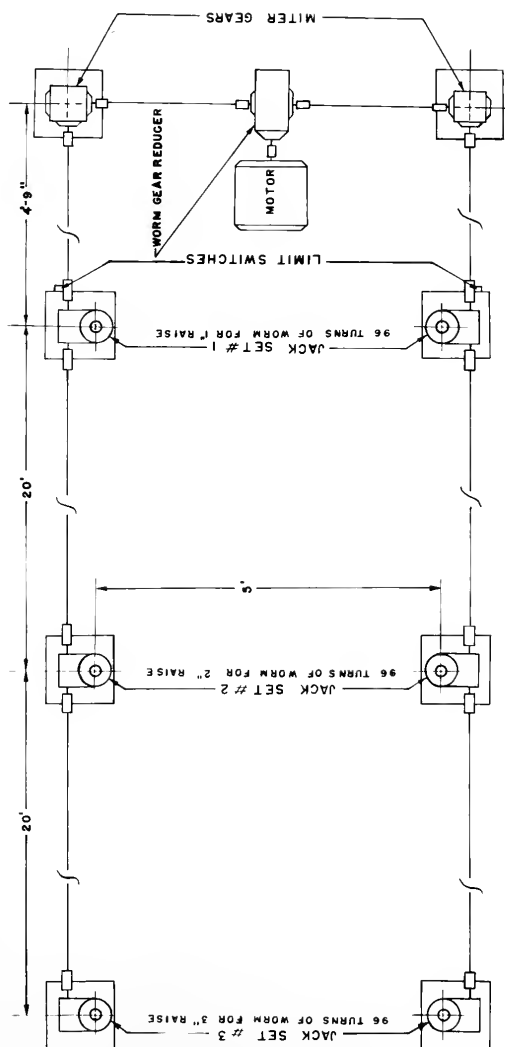
FIGURE 7

FIGURE 8

JACK DETAIL

SLOPE CALIBRATION
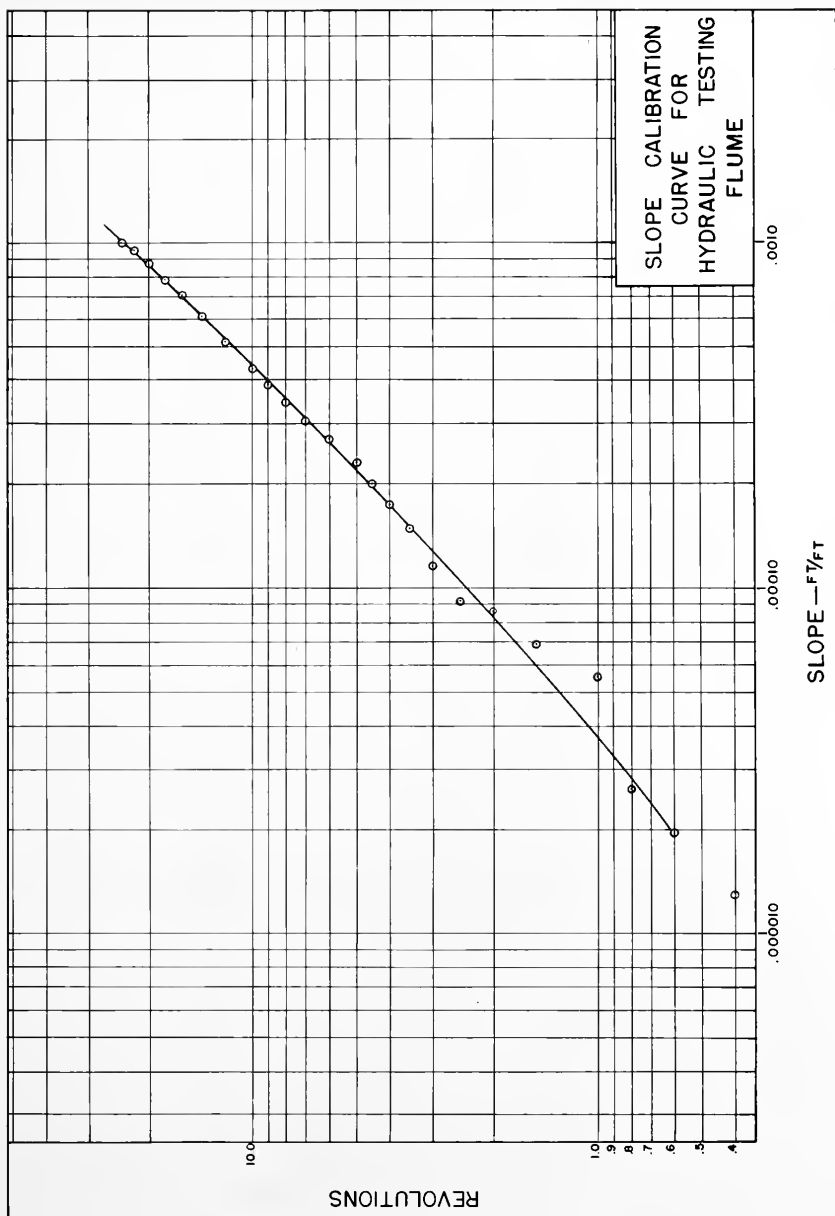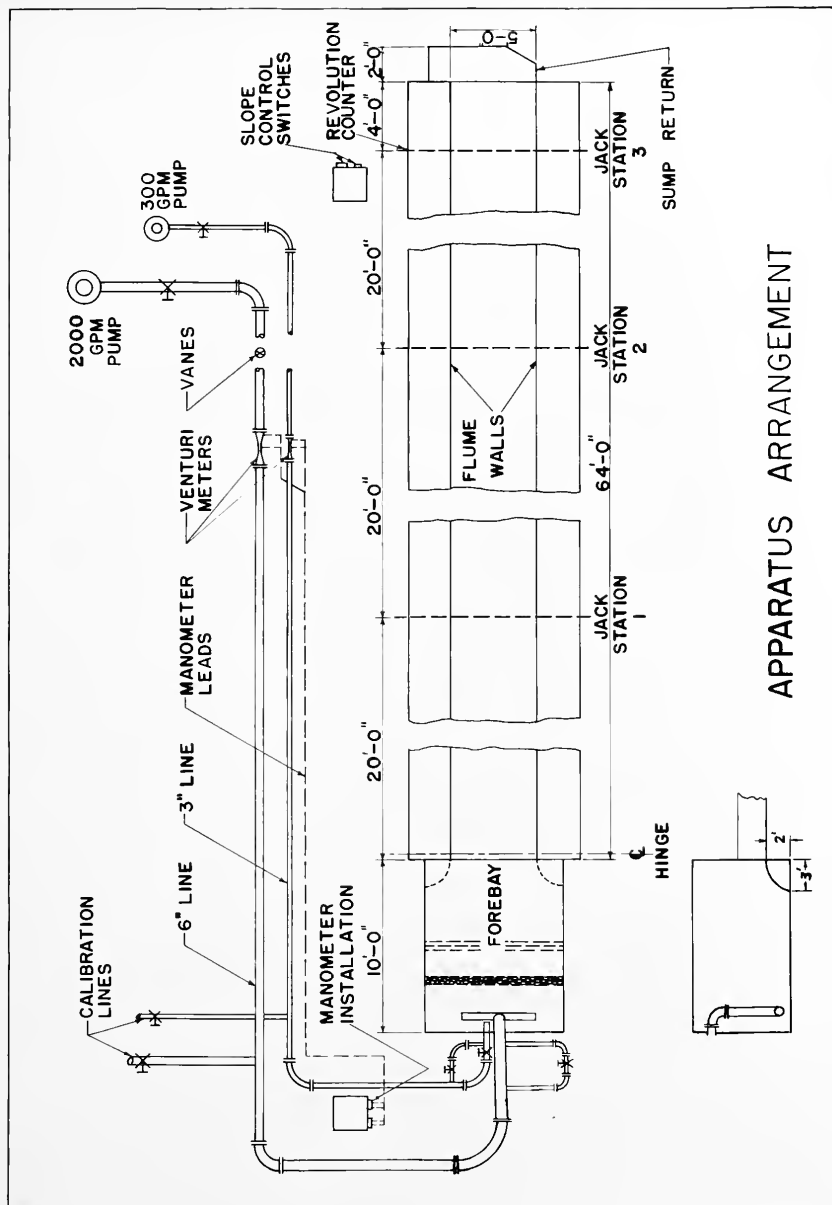CURVE FOR
HYDRAULIC TESTING
FLUME

FIGURE 9

SLOPE—FT/FT

REVOLUTIONS
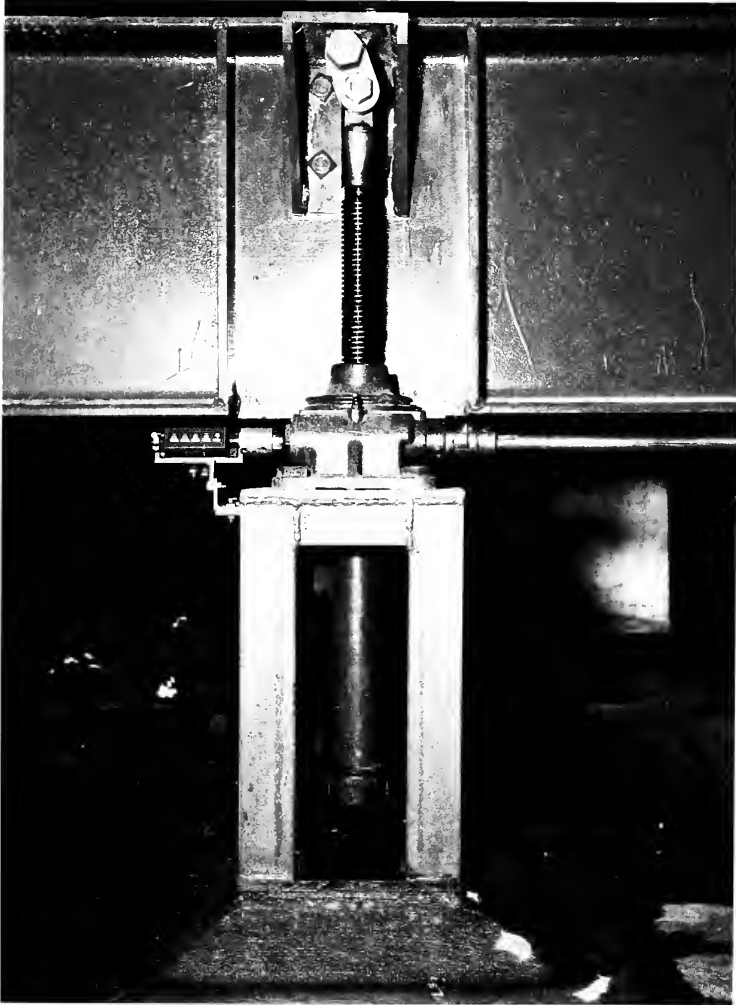
APPARATUS ARRANGEMENT

FIGURE 10

CALIBRATION CURVE FOR
THREE-INCH VENTURI

MANOMETER DEFLECTION; IN. TETRABROMOETHANE, SP. GR. = 2.95

FLOW RATE; CU. FT. PER SEC.

FIGURE 11

CALIBRATION CURVE FOR

SIX-INCH VENTURI

FLOW RATE; CU. FT. PER SEC.

MANOMETER DEFLECTION; IN. MERCURY, SP. GR. = 13.6

FIGURE 12

CALIBRATION CURVE
6" VENTURI

MANOMETER DEFLECTION
IN. TETRABROMOETHANE
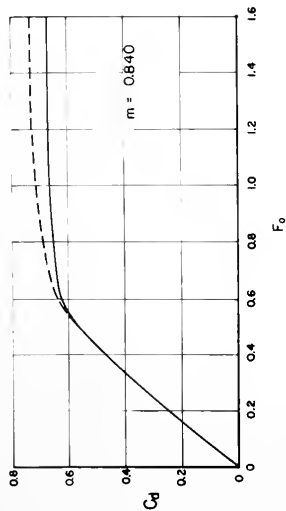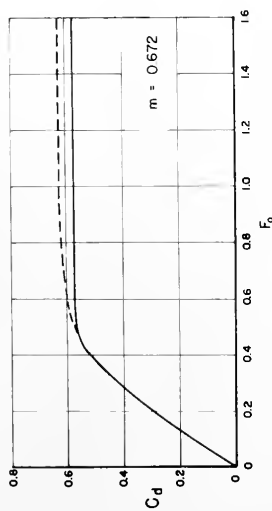SP. GR. = 2.95

FLOW RATE; CU. FT. PER SEC.
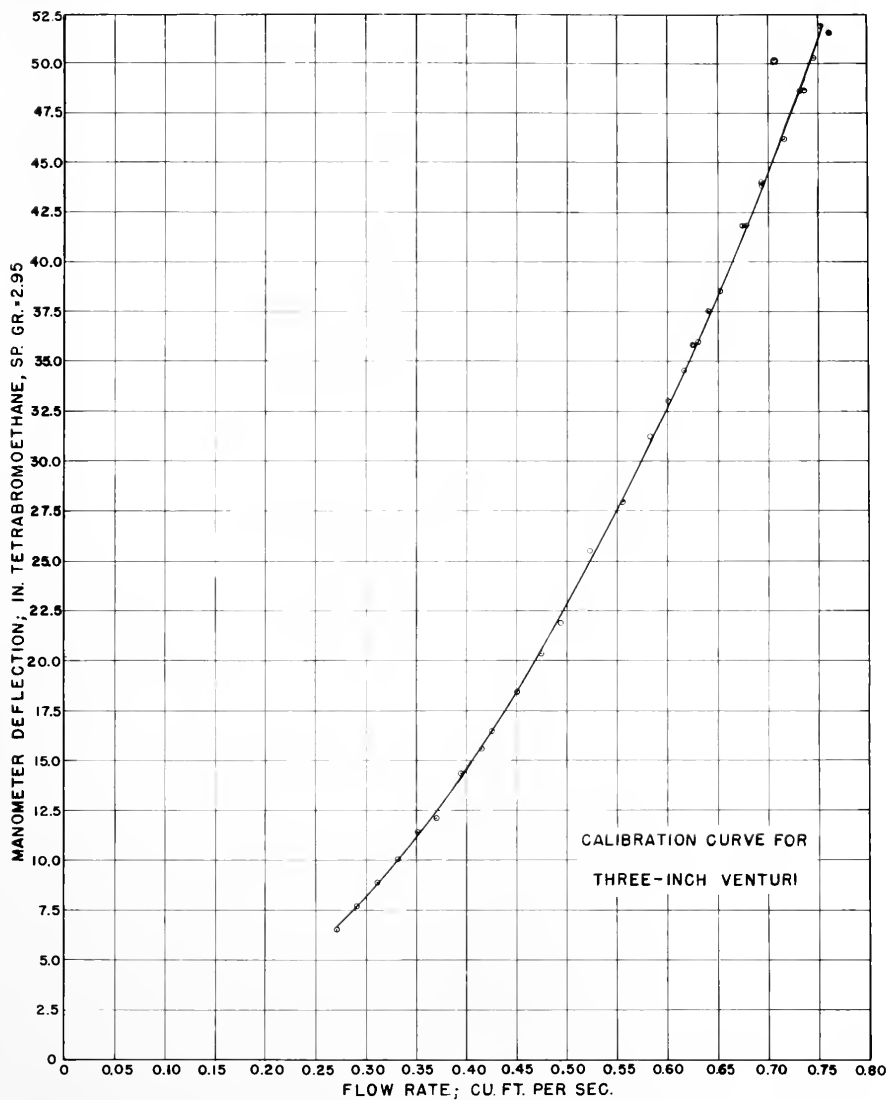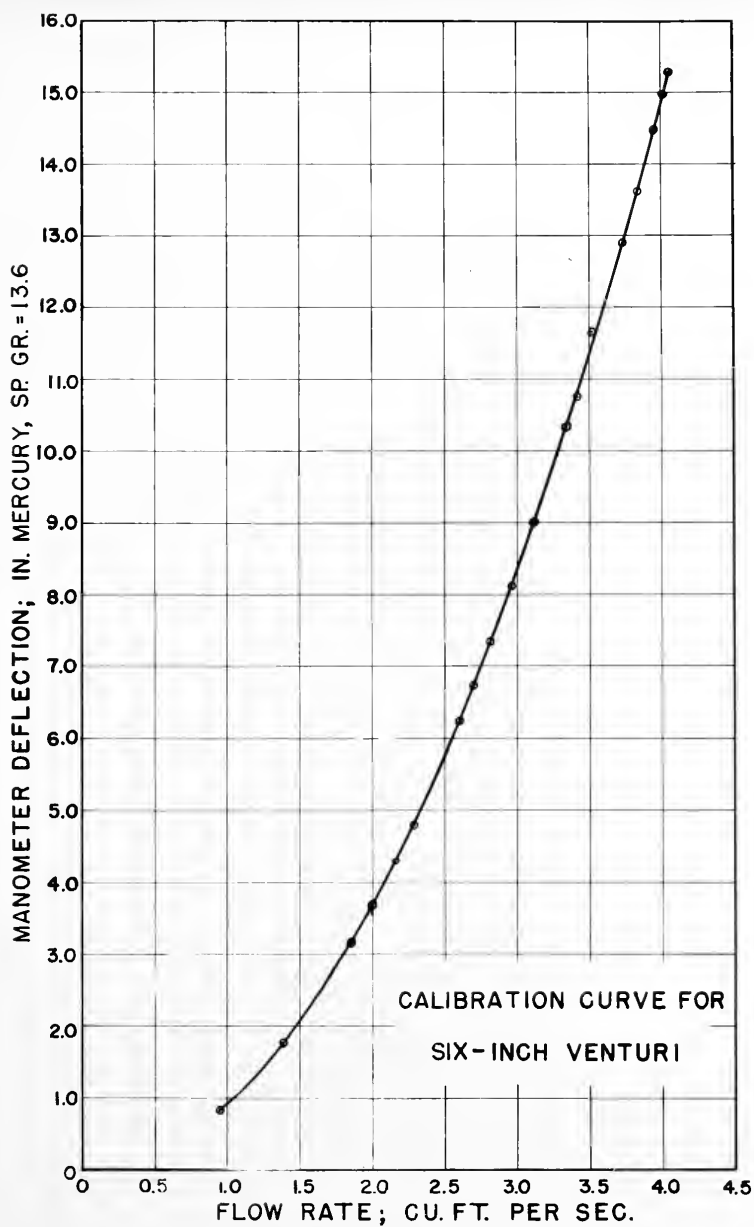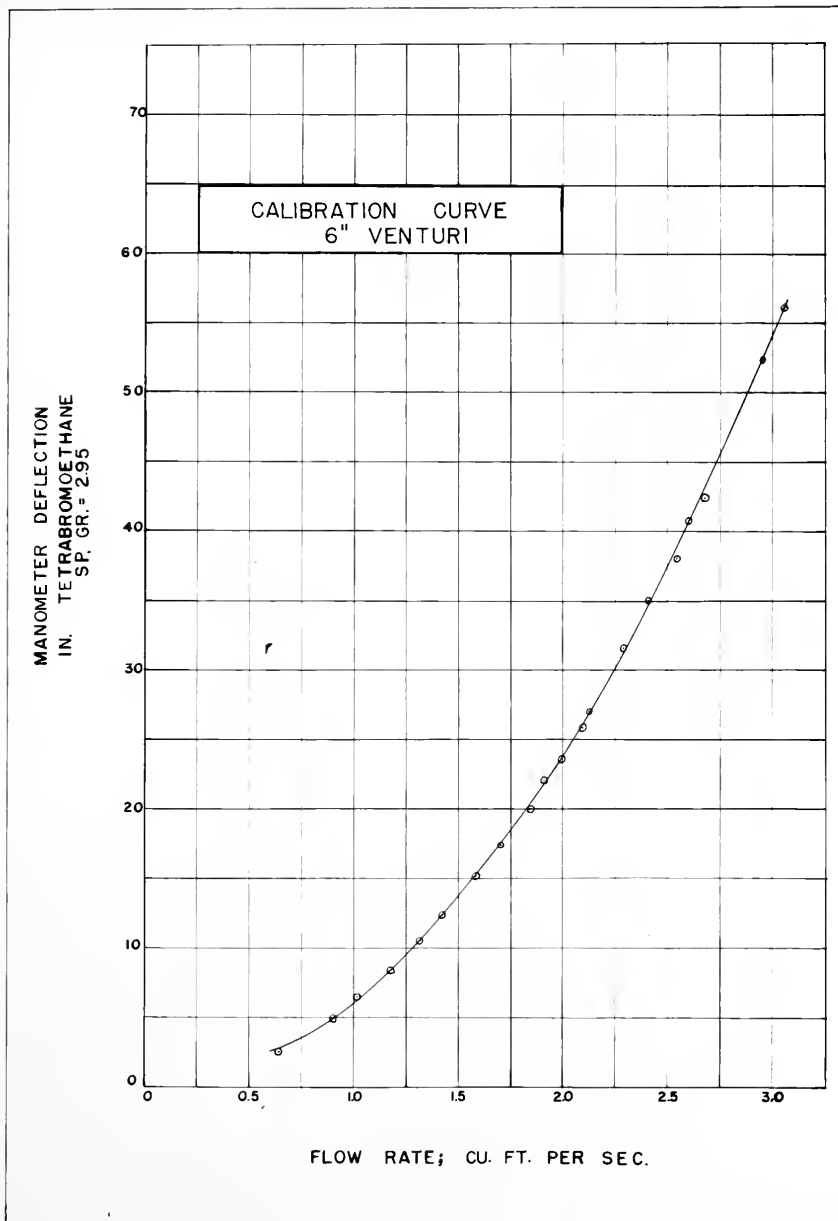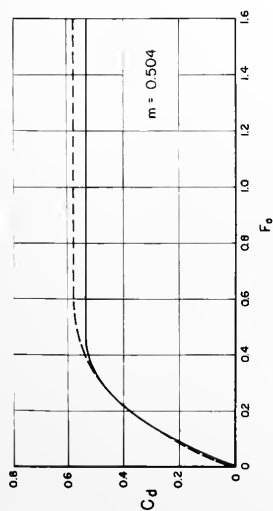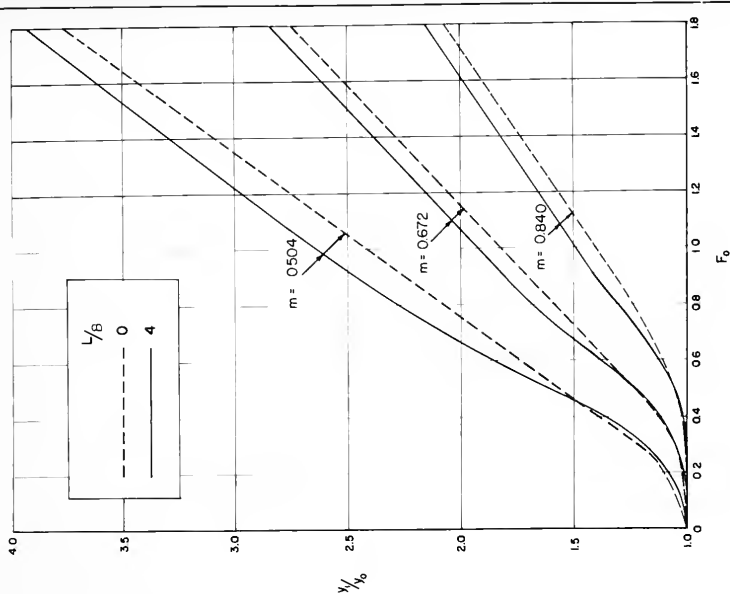
FIGURE 13

FLOW IN RECTANGULAR CHANNELS WITH
SEMI-CIRCULAR CONSTRICTIONS – COMPARISON
OF TWO AND THREE DIMENSIONAL CASES

FIG. 14

SYMMETRICAL
ABOUT ℄

22 GAGE G.I.

STEEL ANGLE

PLYWOOD

FASTENERS

1'-9"

B/2

b/2

b/2

A

A

SEC AA

MODEL
CONSTRUCTION
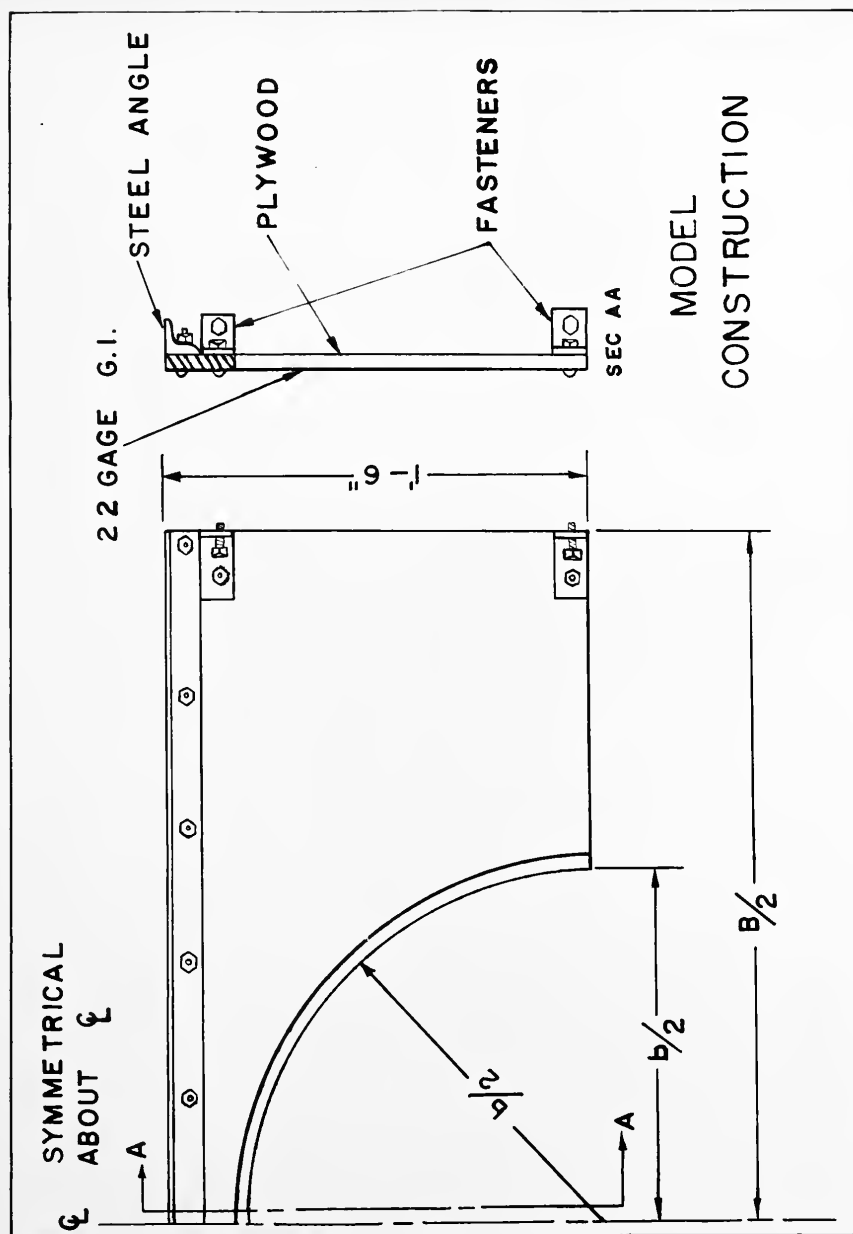
FIGURE 15

FIGURE 16

INSTRUMENT CARRIAGE

FIGURE 17

MODEL IN PLACE

FIGURE 18

MODEL IN PLACE

FIGURE 19

MODEL IN PLACE

FIGURE 21

SLOPE FT./FT.

SLOPE VS. GATE HEIGHT
FOR
TESTING FLUME

GATE HEIGHT CM.

4 CFS
3 CFS
2 CFS

FIGURE 20

SLOPE FT./FT.

NORMAL DEPTH VS. SLOPE
FOR
TESTING FLUME

NORMAL DEPTH Y₀ FT.

4 CFS
3 CFS
2 CFS

SUPERELEVATION
vs
KINETICITY

⊙ – m = .5
△ – m = .7
⊡ – m = .9
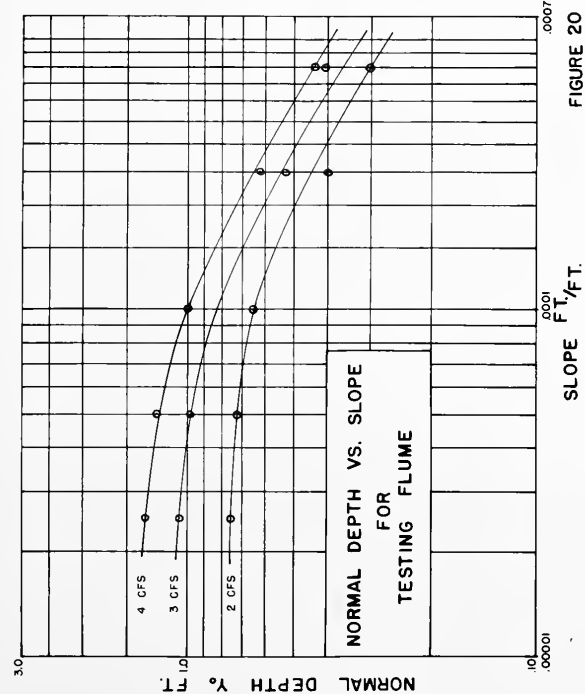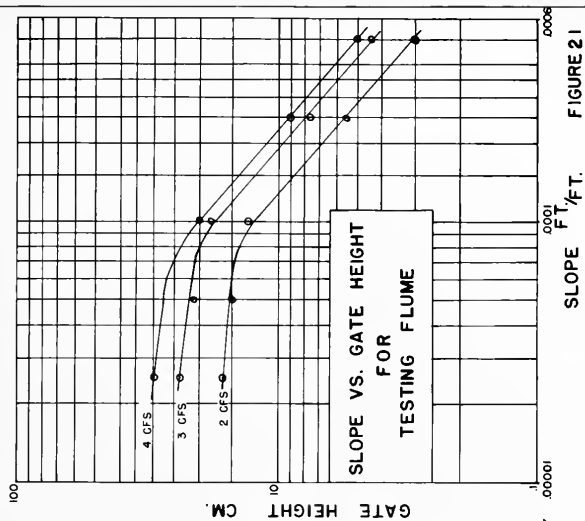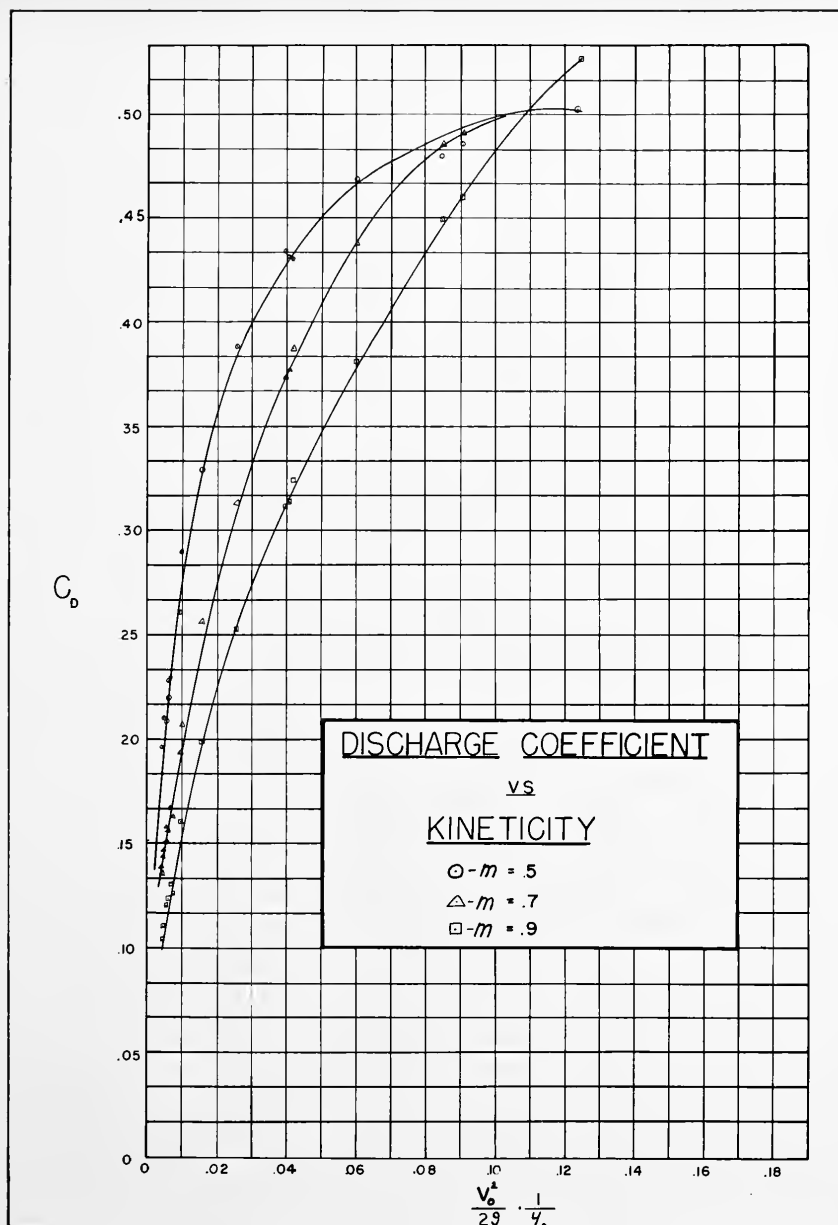
FIGURE 22

FIGURE 23

FIGURE 24

FIGURE 25

FRICTION FACTOR VS REYNOLDS NUMBER

OWEN LINE — WIDE OPEN CHANNELS

$$R = \frac{V \, y}{\nu}$$

$$f = \frac{894 S}{V^2}$$

□   2.0 CFS
◁   3.0 CFS
●   3.5 CFS
⊙   4.0 CFS

BACKWATER RATIO
vs
CONTRACTION RATIO
⊙ −2 CFS
▢ −3 CFS
△ −4 CFS

$n = .013$

$\dfrac{h^*}{\Delta h}$

m
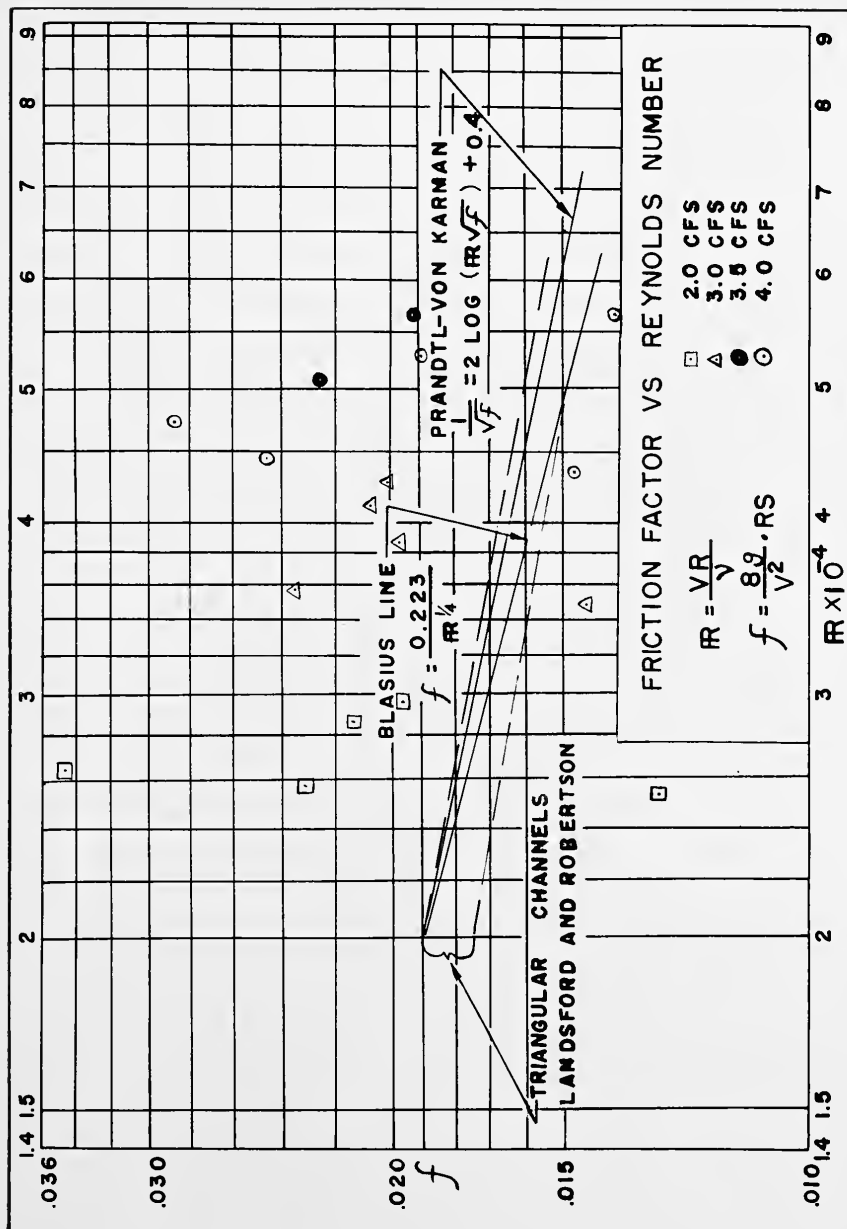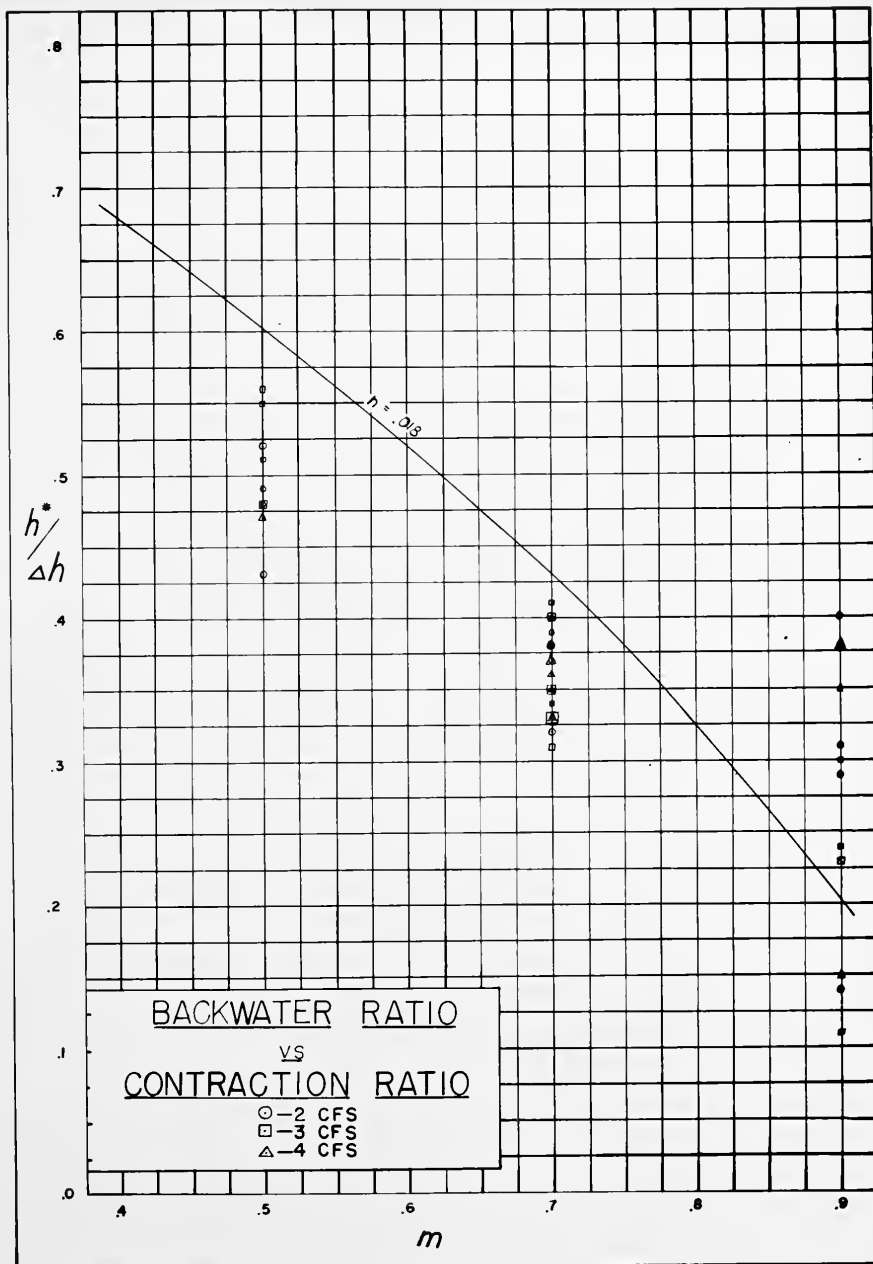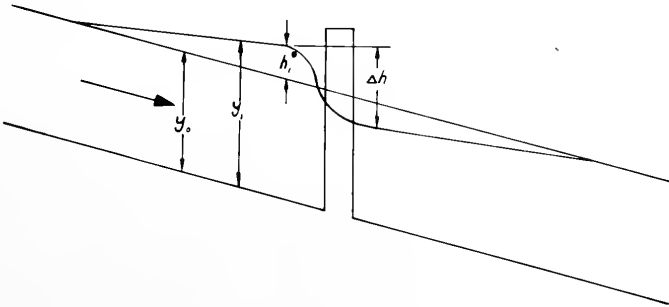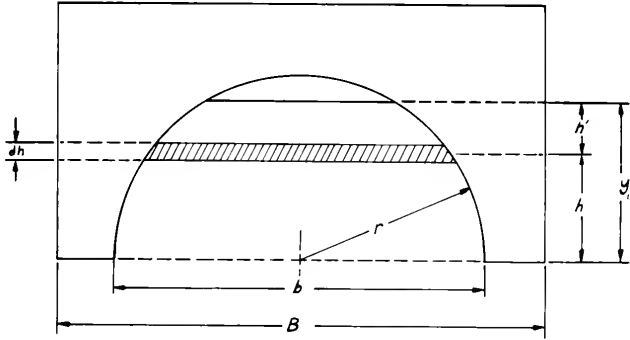
FIGURE 26

DEFINITION SKETCH

FIGURE 27

TABLE 1

OBSERVED AND CALCULATED DATA

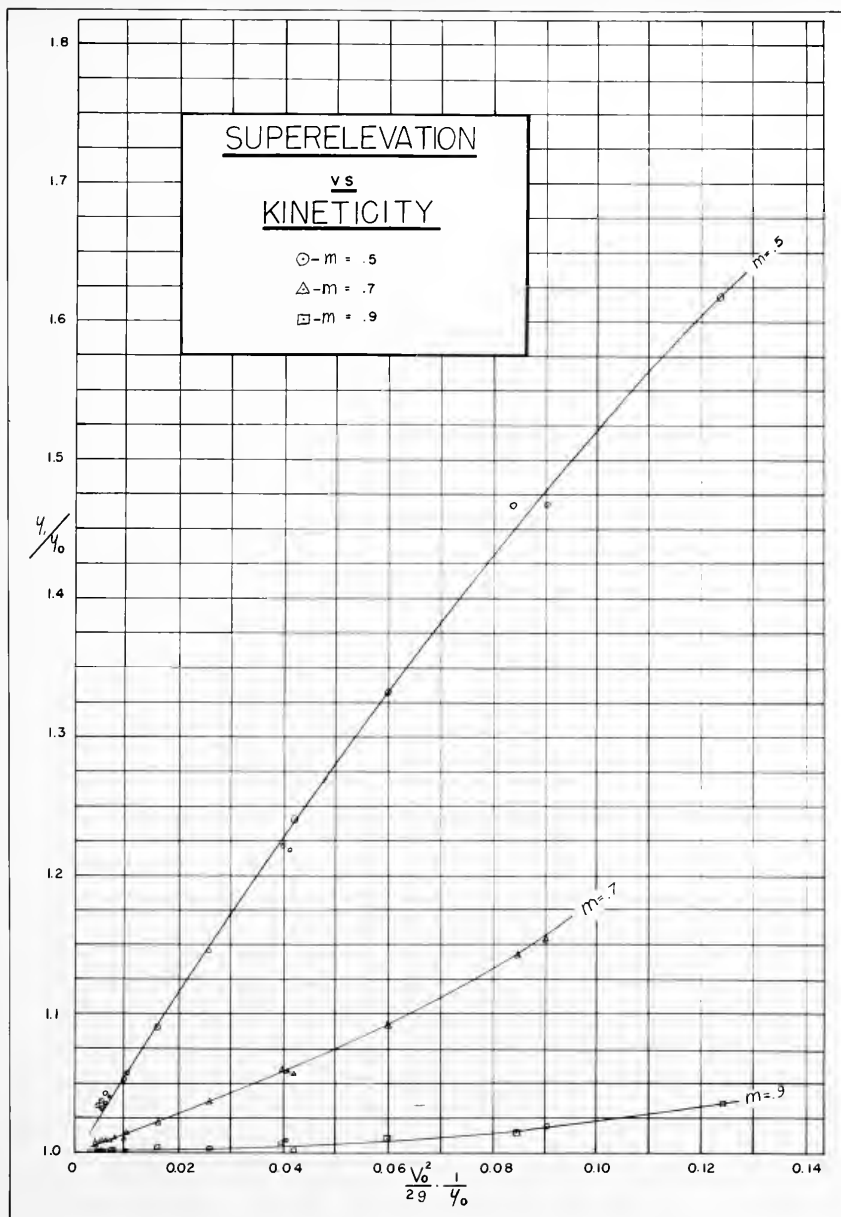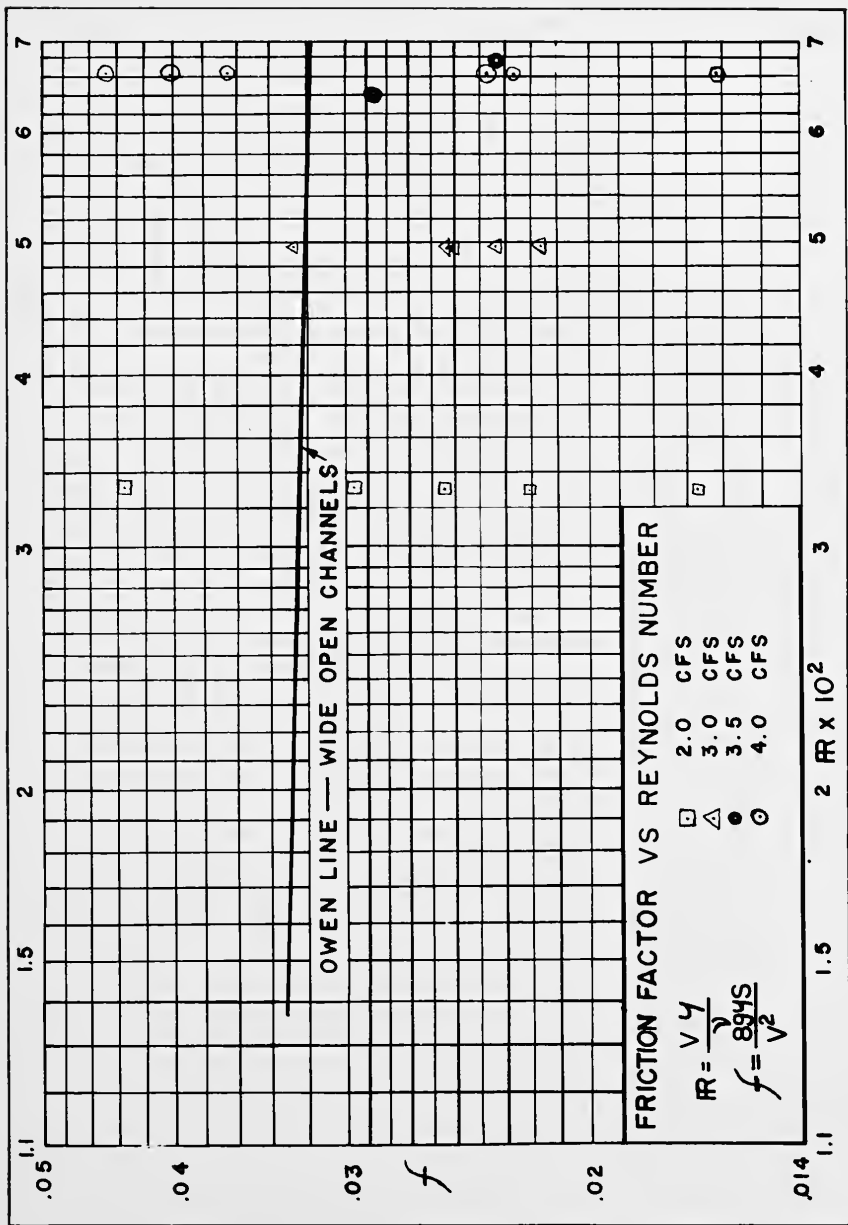| Contraction Ratio	Slope z_{10}^{+6} ft/ft	d cfs	Y_0 cm	Y_1 max cm	Y_1 min cm	R ft	$m=\frac{VR}{v}$	Cd Appendix 2 Equation 2	f	n x10	h_2/h	$V^2/2gY_0$
0.5	0	2	23.78	24.53	22.97	.5934	25,147	.2087	---	---	.48	.0053
		3	33.97	35.18	32.97	.7688	34,213	.1962	---	---	.55	.0041
	25	2	22.88	23.70	22.03	.5762	25,377	.2190	.0128	.0957	.49	.0060
		3	32.15	33.33	31.17	.7400	34,795	.2095	.0145	.1060	.55	.0048
	50	3	22.02	22.90	21.11	.5584	25,606	.2294	.0231	.1276	.49	.0067
		3	29.83	31.10	28.60	.7017	35,565	.2282	.0236	.1341	.51	.0060
	100	3	19.86	20.90	18.73	.5160	26,188	.2607	.0347	.1544	.48	.0091
		3	21.70	23.66	20.19	.5531	38,533	.3288	.0197	.1177	.56	.0158
	185	3	30.73	32.73	28.82	.7167	47,008	.4318	.0288	.1489	.51	.0099
		3.5	20.45	23.43	17.70	.5280	50,555	.4329	.0227	.1255	.52	.0407
	250	3	12.07	14.72	9.61	.3415	28,516	.3807	.0212	.1127	.48	.0397
		3	15.94	19.45	12.14	.4319	40,959	.4800	.0208	.1161	.43	.0419
	350	3.5	18.97	23.53	13.00	.4975	52,860	.4803	.0191	.1138	.42	.0904
		3	9.25	13.57	8.34	.2704	56,385	.5026	.0197	.1115	.55	.0842
	500	2	12.38	13.57	7.67	.3490	29,459	.3888	.0202	.1046	.47	.1233
		3	13.24	21.45	4.04	.3696	56,270	.4685	.0138	.0920	.32	.0256
0.7	0	2	33.97	34.18	33.57	.5934	34,213	.1506	---	---	.36	.0060
		3	41.46	41.76	40.93	.7688	42,697	.1357	---	---	.40	.0053
	25	2	22.88	23.08	22.58	.5762	25,377	.1385	.0128	.0957	.36	.0041
		3	32.15	32.34	31.72	.7400	34,795	.1568	.0145	.1060	.31	.0060
	50	3	40.14	40.41	39.63	.8600	43,180	.1466	.0148	.1114	.35	.0048
		2	22.02	22.22	21.71	.5584	25,606	.1446	.0231	.1276	.39	.0044
	100	3	29.75	30.06	29.31	.7003	35,590	.1668	.0235	.1319	.38	.0067
		3	37.50	37.82	36.97	.8222	44,197	.1625	.0246	.1406	.33	.0075
	185	3	19.86	20.07	19.43	.5160	26,188	.1579	.0347	.1544	.38	.0054
		3	21.70	22.18	20.98	.5531	38,533	.1934	.0197	.1177	.33	.0091
	250	3	30.69	31.13	29.95	.7161	47,023	.2560	.0287	.1481	.40	.0158
		3.5	20.45	21.19	19.13	.5280	50,555	.2062	.0227	.1281	.37	.0099
	350	3.5	12.07	12.78	10.84	.3415	28,516	.3765	.0212	.1127	.36	.0407
		3	15.94	16.89	14.19	.4319	40,959	.3737	.0208	.1161	.37	.0397
	500	3	18.97	20.06	16.80	.4975	52,860	.3870	.0191	.1138	.35	.0419
		3	9.25	16.84	13.20	.2704	56,385	.4913	.0197	.1078	.33	.0904
		2	12.38	10.68	4.94	.5584	42,615	.4858	.0197	.1046	.38	.0848
		3	13.24	8.75	8.75	.7003	25,385	.3127	.0202	.1107	.11	.0256
0.9	0	2	33.97	33.99	33.81	.5531	25,395	.1038	.0128	.0955	.15	.0600
		3	40.14	40.17	39.81	.5756	43,180	.1233	.0148	.1114	.29	.0041
	25	2	22.02	22.06	21.92	.8600	25,606	.1095	.0231	.1276	.23	.0044
		3	29.75	29.78	29.65	.7003	35,590	.1299	.0235	.1319	.33	.0067
	50	3	37.50	37.57	37.37	.8222	44,197	.1257	.0246	.1406	.35	.0075
		3	21.70	21.78	21.57	.5531	38,533	.1202	.0197	.1177	.19	.0054
	100	3	30.78	30.78	30.51	.7167	47,008	.1984	.0288	.1489	---	.0099
		3.5	20.45	23.43	20.22	.5280	50,555	.1597	.0227	.1255	.40	.0407
	185	3	12.07	12.17	11.94	.3415	28,516	.3139	.0208	.1127	.04	.0419
		3	15.94	16.05	15.68	.4319	40,959	.3239	.0208	.1161	.21	.0993
	250	3.5	18.97	18.99	18.52	.4975	52,860	.4599	.0191	.1161	.14	.0419
		3	9.25	9.42	4.87	.2704	56,270	.4490	.0194	.2855	.27	.1224
	350	3.5	15.40	15.56	11.81	.4975	29,459	.5282	.0197	.1045	.14	.0256
		3	12.38	12.56	1.21	.3490	42,615	.2526	.0202	.1107	---	.0256
	500	3	13.20	13.66	1.21	.3867	56,295	.3908	.0137	.1008	.38	.0600